**University of Technology Sydney**
**School of Mathematical and Physical Sciences**

Mathematical Statistics (37262) –
Tutorial 10
SOLUTIONS

1. i)

| State | Proposal | Acceptance Probability | $A_j > u_{10+j}$? | Accept Move? |
|---|---|---|---|---|
| $x_0 = 4$ | 1 | $A_1 = \min\left\{1, \frac{0.7}{0.7(0.3)^3}\right\} = 1$ | $1 > 0.503$ | Yes |
| $x_1 = 1$ | 2 | $A_2 = \min\left\{1, \frac{0.7(0.3)}{0.7}\right\} = 0.3$ | $0.3 > 0.111$ | Yes |
| $x_2 = 2$ | -1 | $A_3 = \min\left\{1, \frac{0}{0.7(0.3)}\right\} = 0$ | $0 < 0.859$ | No |
| $x_3 = 2$ | 1 | $A_4 = \min\left\{1, \frac{0.7}{0.7(0.3)}\right\} = 1$ | $1 > 0.330$ | Yes |
| $x_4 = 1$ | 5 | $A_5 = \min\left\{1, \frac{0.7(0.3)^4}{0.7}\right\} = 0.0081$ | $0.0081 < 0.474$ | No |
| $x_5 = 1$ | 0 | $A_6 = \min\left\{1, \frac{0}{0.7}\right\} = 0$ | $0 < 0.004$ | No |
| $x_6 = 1$ | 3 | $A_7 = \min\left\{1, \frac{0.7(0.3)^2}{0.7}\right\} = 0.09$ | $0.09 < 0.217$ | No |
| $x_7 = 1$ | 2 | $A_8 = \min\left\{1, \frac{0.7(0.3)}{0.7}\right\} = 0.3$ | $0.3 < 0.712$ | No |
| $x_8 = 1$ | 3 | $A_9 = \min\left\{1, \frac{0.7(0.3)^2}{0.7}\right\} = 0.09$ | $0.09 < 0.557$ | No |
| $x_9 = 1$ | -1 | $A_{10} = \min\left\{1, \frac{0}{0.7}\right\} = 0$ | $0 < 0.486$ | No |
| $x_{10} = 1$ | | | | |

ii) A normal random variable $Y \sim N(\mu, \sigma^2)$ would not be suitable as a proposal distribution regardless of the choice of $\mu$ and $\sigma^2$ since it is a continuous distribution, whereas the geometric variable is discrete and only takes integer values.

iii) In this case, the acceptance rate would be 0.6, since all proposals of moves to 1, 2 or 3 would be accepted (acceptance probability of 1) and all proposals of moves to 0 or 4 would be rejected (acceptance probability of 0) hence the overall rate would be 60% or 0.6.

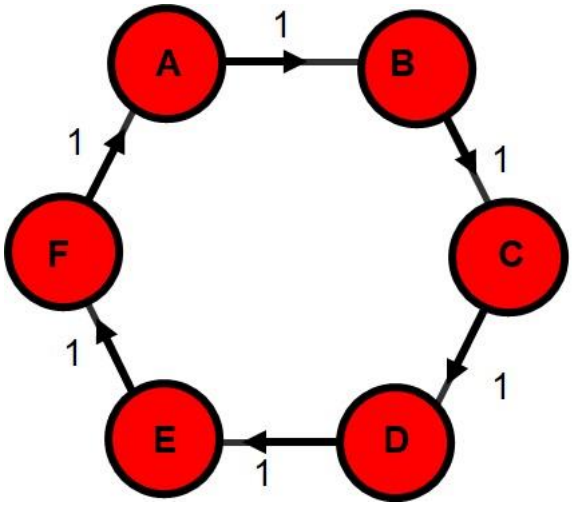
2. i)

| State | Proposal | Acceptance Probability | $A_j > u_{10+j}$? | Accept Move? |
|---|---|---|---|---|
| $x_0 = 4$ | 1 | $A_1 = \min\left\{1, \frac{0.1}{0.1}\right\} = 1$ | $1 > 0.503$ | Yes |
| $x_1 = 1$ | 2 | $A_2 = \min\left\{1, \frac{0.1}{0.1}\right\} = 1$ | $1 > 0.111$ | Yes |
| $x_2 = 2$ | -1 | $A_3 = \min\left\{1, \frac{0.1}{0.1}\right\} = 1$ | $1 > 0.859$ | Yes |
| $x_3 = -1$ | 1 | $A_4 = \min\left\{1, \frac{0.1}{0.1}\right\} = 1$ | $1 > 0.330$ | Yes |
| $x_4 = 1$ | 5 | $A_5 = \min\left\{1, \frac{0}{0.1}\right\} = 0$ | $0 < 0.474$ | No |
| $x_5 = 1$ | 0 | $A_6 = \min\left\{1, \frac{0.2}{0.1}\right\} = 1$ | $1 > 0.004$ | Yes |
| $x_6 = 0$ | 3 | $A_7 = \min\left\{1, \frac{0.2}{0.2}\right\} = 1$ | $1 > 0.217$ | Yes |
| $x_7 = 3$ | 2 | $A_8 = \min\left\{1, \frac{0.1}{0.2}\right\} = 0.5$ | $0.5 < 0.712$ | No |
| $x_8 = 3$ | 3 | $A_9 = \min\left\{1, \frac{0.2}{0.2}\right\} = 1$ | $1 > 0.557$ | Yes |
| $x_9 = 3$ | -1 | $A_{10} = \min\left\{1, \frac{0.1}{0.2}\right\} = 0.5$ | $0.5 > 0.486$ | Yes |
| $x_{10} = -1$ | | | | |

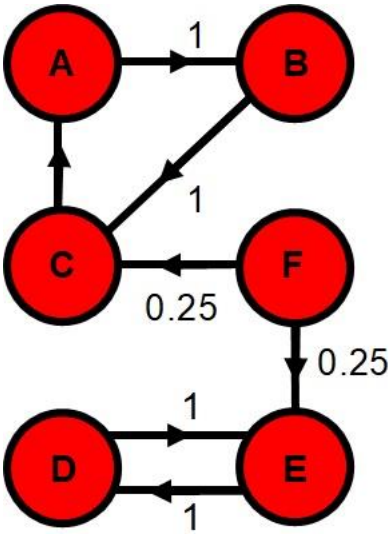
ii) A Poisson variable not be suitable as a proposal distribution regardless of the choice of rate parameter since the Poisson only takes non-negative values hence cannot propose all possible values from the target distribution.

3. Many different answers for each. Arrows from each state must include probability weights and not exceed probability 1 of leaving any state. Once such example for each is given below.

i) The Markov Chain is ergodic.




ii) States A, B and C each have period 3 and states D and E have period 2.




iii) The system has two equilibrium distributions, either being in A or B with equal probability or in C or D with equal probability. Starting in E it will always end up in the A or B equilibrium and starting in F it may end up in either of the two equilibria.