University of Technology Sydney
School of Mathematical and Physical Sciences

Mathematical Statistics (37262) –
Tutorial 2

1. Let Z be a continuous random variable with probability density function

$$f(z) = \begin{cases} (z-1)^3 & 1 < z < 1+\sqrt{2} \\ 0 & \text{otherwise} \end{cases}.$$

i) Calculate the cumulative probability function of Z, $F(z)$.

ii) Show that $F^{-1}(z)$, the inverse of the cumulative probability function is given by $F^{-1}(z) = 1 + \sqrt[4]{4z}$. .

iii) Given realisations $\{u_1, u_2, ..., u_5\} = \{0.710,\ \ 0.119,\ \ 0.358,\ \ 0.883,\ \ 0.504\}$ of a $U[0,1]$ variable, generate five realisations $\{z_1, z_2, ..., z_5\}$ of Z

Clearly explain your method and any calculations required.

2. Let Z be a continuous random variable with probability density function

$$f(z) = \frac{\sin(z)}{2} \text{ for } z \in [0, \pi).$$

i) Show that the cumulative probability function of Z, $F(z)$ is given by

$$F(z) = \begin{cases} 0 & z < 0 \\ \dfrac{1-\cos(z)}{2} & z \in [0, \pi) \\ 1 & z \geq \pi \end{cases}.$$

ii) Given realisations $\{u_1, u_2, ..., u_5\} = \{0.710,\ 0.119,\ 0.358,\ 0.883,\ 0.504\}$ of a $U[0,1]$ variable, generate five realisations $\{z_1, z_2, ..., z_5\}$ of Z.

Clearly explain your method and any calculations required.

3. Let Y be a continuous uniform random variable, $Y \sim Gumbel(\mu, \beta)$.for $\beta > 0$.

That is, Y has cumulative probability function

$$P(Y \leq y) = F(y) = e^{-e^{-\left(\frac{y-\mu}{\beta}\right)}} \text{ for } y \in \mathbb{R}.$$

i) Showing all of your working, find the probability density function of Y.

ii) Show that the inverse of the cumulative probability function is given by

$$F^{-1}(y) = \mu - \beta \ln(-\ln(y)). \text{ for } y \in \mathbb{R}.$$

iii) Given realisations $\{u_1, u_2, ..., u_5\} = \{0.710, 0.119, 0.358, 0.883, 0.504\}$ of a $U[0,1]$ variable, generate five realisations $\{y_1, y_2, ..., y_5\}$ of $Y \sim Gumbel(5,10)$.

Clearly explain your method and any calculations required.