University of Technology Sydney
School of Mathematical and Physical Sciences

Mathematical Statistics (37262) –
Tutorial 2 – Assessable Labwork

1. A realisation x_i of a random variable *X* is generated according the rule

   If $u_i < 0.1$ then $x_i = 0.1$
   If $u_i > 0.8$ then $x_i = 1$
   Otherwise, $x_i = 0$

   where u_i is a realisation of a uniform random variable $U \sim U[0,1]$.

   i) Calculate the probability mass function of *X*.

   ii) Hence show that $E(X) = 0.21$.

   Let *Y* be a continuous random variable with probability density function

   $$f(y) = \begin{cases} 1.5y^2 & y \in [-1,1] \\ 0 & \text{otherwise} \end{cases}.$$

   iii) Write down $E(Y)$. Justify your answer.

   iv) Find the cumulative distribution of *Y*, $F(y) = P(Y \le y)$.

   v) Hence show that $F^{-1}(y_i) = \sqrt[3]{2y-1}$.

   A realisation of *Y* is generated by setting $y_i = F^{-1}(u_i)$ where u_i is a realisation of $U \sim U[0,1]$.

   Let $Z = \max\{X,Y\}$ be a random variable taking either the value of *X* or the value of *Y*, whichever is greater, when *X* and *Y* are generated as above, using the same uniform realisations.

   vi) Using the values $\{u_1,u_2,u_3\} = \{0.402,\ 0.009,\ 0.711\}$, generate realisations $\{z_1,z_2,z_3\}$ of *Z*.

   vii) Calculate $P(Z > 0)$. Justify your answer.