University of Technology Sydney
School of Mathematical and Physical Sciences

Mathematical Statistics (37262) – Tutorial 2 SOLUTIONS

1. i) For $z \leq 1$, the cumulative probability is 0. For $z \geq 1+\sqrt{2}$, the cumulative probability is 1. Between these values, we have

$$F(z)=\int_1^z f(t)dt=\left[\frac{(t-1)^4}{4}\right]_1^z .$$

Hence $$F(z)=\begin{cases} 0 & z \leq 1 \\ \dfrac{(z-1)^4}{4} & 1<z<1+\sqrt{2} \\ 1 & z \geq 1+\sqrt{2} \end{cases}$$

ii) For $F(z)=\dfrac{(z-1)^4}{4}$ we have $4F(z)=(z-1)^4$ and hence

$z=1+\sqrt[4]{4F(z)}$.

This therefore gives the inverse function $F^{-1}(z)=1+\sqrt[4]{4z}$.

iii) Realisations $\{z_1,z_2,...,z_5\}$ of Z can be obtained by calculating

$z_i=1+\sqrt[4]{4u_i}$ where each u_i is a realisation of a $U[0,1]$ variable.

For $\{u_1,u_2,...,u_5\}=\{0.710,\ 0.119,\ 0.358,\ 0.883,\ 0.504\}$, we obtain

$\{z_1,z_2,...,z_5\}\approx\{2.298,\ 1.831,\ 2.094,\ 2.371,\ 2.192\}$.

2.

i) $F(z)=\int_0^z \frac{\sin(t)}{2}dt$ for $z\in[0,\pi)$,

$\int_0^z \frac{\sin(t)}{2}dt\left[\frac{-\cos(t)}{2}\right]_0^z=\frac{1-\cos(z)}{2}$ for $z\in[0,\pi)$.

Hence $F(z)=\begin{cases}0 & z<0\\ \frac{1-\cos(z)}{2} & z\in[0,\pi)\\ 1 & z\geq\pi\end{cases}$.

ii) Given u_i, $z_i=\arccos(1-2u_i)$ hence for

$\{u_1,u_2,...,u_5\}=\{0.710,\ 0.119,\ 0.358,\ 0.883,\ 0.504\}$,

$\{z_1,z_2,...,z_5\}=\{2.004,\ 0.704,\ 1.283,\ 2.443,\ 1.579\}$

3.

i) $P(Y\leq y)=F(y)=e^{-e^{-\left(\frac{y-\mu}{\beta}\right)}}$ for $y\in\mathbb{R}$.

$$f(y)=\frac{dF(y)}{dy}=\frac{d}{dy}\left(e^{-e^{-\left(\frac{y-\mu}{\beta}\right)}}\right)=\left(\frac{1}{\beta}e^{-\left(\frac{y-\mu}{\beta}\right)}\right)\left(e^{-e^{-\left(\frac{y-\mu}{\beta}\right)}}\right)\text{ for } y\in\mathbb{R}.$$

ii) If $F(y)=e^{-e^{-\left(\frac{y-\mu}{\beta}\right)}}$ then $-\ln(F(y))=e^{-\left(\frac{y-\mu}{\beta}\right)}$.

This gives $-\ln(-\ln(F(y)))=\left(\frac{y-\mu}{\beta}\right)$ and hence

$\mu-\beta\ln(-\ln(F(y)))=y$.

Setting when $y=F^{-1}(y)$ to find the inverse, we obtain

$F^{-1}(y)=\mu-\beta\ln(-\ln(y))$. for $y\in\mathbb{R}$.

i) This gives that, if u_i is a realisation of a $U[0,1]$ variable, then

$y_i=5-10\ln(-\ln(u_i))$ is a realisation of $Y\sim Gumbel(5,10)$.

$u_1=0.710$ hence $y_1=5-10\ln(-\ln(0.710))\approx 15.71$
$u_2=0.119$ hence $y_2=5-10\ln(-\ln(0.119))\approx -2.55$
$u_3=0.358$ hence $y_3=5-10\ln(-\ln(0.358))\approx 4.73$
$u_4=0.883$ hence $y_4=5-10\ln(-\ln(0.883))\approx 25.84$
$u_5=0.504$ hence $y_5=5-10\ln(-\ln(0.504))\approx 8.78$