University of Technology Sydney
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Mathematical Statistics (37262) –
Tutorial 3

1. Consider two discrete random variables X and Y with joint probability mass function

$$P((X,Y)=(m,n))=\begin{cases} 0.1 & (m,n)=(0,0) \\ 0.02 & (m,n)=(0,-1) \\ 0.02 & (m,n)=(0,1) \\ 0.05 & (m,n)=(-1,-1) \\ 0.01 & (m,n)=(1,-1) \\ 0.01 & (m,n)=(-1,1) \\ 0.1 & (m,n)=(1,1) \\ 0.2 & (m,n)=(-1,2) \\ 0.05 & (m,n)=(0,2) \\ 0.15 & (m,n)=(1,2) \\ 0.09 & (m,n)=(0,-2) \\ 0.05 & (m,n)=(-1,-2) \\ 0.075 & (m,n)=(1,-2) \\ 0.075 & (m,n)=(0,3) \\ 0 & \text{otherwise} \end{cases}$$

i) Calculate the marginal probability mass function of X, $f(X)=P(X=m)$.

ii) Calculate the marginal probability mass function of Y, $f(Y)=P(Y=n)$.

iii) Show that the conditional probability mass function of X given $X=Y$ is

$$P(X=m \mid X=Y)=\begin{cases} 0.4 & m=0 \\ 0.2 & m=-1 \\ 0.4 & m=1 \\ 0 & \text{otherwise} \end{cases}.$$

iv) Calculate the conditional probability mass function of Y given $X=-1$.

v) Write down the conditional probability mass function of X given $Y=3$.

2. Consider two continuous random variables X and Y with joint probability density function

$$f_{X,Y}(x,y) = \begin{cases} \dfrac{2xy+1}{18} & 1 < x < y < 3 \\ 0 & \text{otherwise} \end{cases}$$

i) Show that the marginal density of X, $f_X(x)$ is

$$f_X(x) = \begin{cases} \dfrac{3+8x-x^3}{18} & 1 < x < 3 \\ 0 & \text{otherwise} \end{cases}.$$

ii) Calculate the marginal density function of Y, $f_Y(y)$.

iii) Show that $P(y - x > 1) = \dfrac{49}{216}$