University of Technology Sydney
School of Mathematical and Physical Sciences

Mathematical Statistics (37262) –
Tutorial 3 – Assessable Labwork

1. Let *X, Y* and *Z* be discrete random variables with joint probability mass function

$$P((X,Y,Z)=(k,l,m))=\begin{cases} 0.2 & (k,l,m)=(0,0,0) \\ 0.1 & (k,l,m)=(0,0,1) \\ 0.12 & (k,l,m)=(0,1,0) \\ 0.18 & (k,l,m)=(0,1,1) \\ 0.05 & (k,l,m)=(1,0,0) \\ 0.15 & (k,l,m)=(1,0,1) \\ 0.1 & (k,l,m)=(1,1,0) \\ 0.1 & (k,l,m)=(1,1,1) \\ 0 & \text{otherwise} \end{cases}$$

i) Calculate the marginal distribution of *X*.

ii) Hence show that the variance of *X* is 0.24,

iii) Calculate the joint marginal distribution of *X* and *Y* and hence show that *X* and *Y* are independent.
That is, show that $P((X,Y)=(k,l))=P(X=k)P(Y=l)$ for all possible values of *k* and *l*.

iv) Are *X* and *Z* independent? Justify your answer.

v) Find the conditional distribution of *X* given $X=Z$.

2. Three continuous random variables have joint density function $f_{X,Y,Z}(x,y,z)$ which is non-zero on the three dimensional interval $(x,y,z) \in [0,\infty)^3$.

Consider the change of variables $R = X + 1$ and $S = \sqrt{Y}$ and $T = Z - Y^2$.

i) Calculate *X*, *Y* and *Z* as functions of *R, S* and *T*.

ii) What are the ranges of *R, S* and *T*.?

iii) Hence show that the absolute value of the Jacobian matrix associated with this change of variables is given by

$$\left|\det(\boldsymbol{J})\right| = \left|\det\begin{pmatrix} \frac{\partial X}{\partial R} & \frac{\partial X}{\partial S} & \frac{\partial X}{\partial T} \\ \frac{\partial Y}{\partial R} & \frac{\partial Y}{\partial S} & \frac{\partial Y}{\partial T} \\ \frac{\partial Z}{\partial R} & \frac{\partial Z}{\partial S} & \frac{\partial Z}{\partial T} \end{pmatrix}\right| = 2S .$$