University of Technology Sydney
School of Mathematical and Physical Sciences

Mathematical Statistics (37262) –
Tutorial 3
SOLUTIONS

1.

i) X has range $\{-1,0,1\}$. We obtain the probability masses by summing appropriate probabilities from the joint probability mass function. For example, $P(X=0)$ is obtained by summing the probabilities that (X,Y) takes the ordered pairs $(0,0),(0,1),(0,-1),(0,2),(0,-2)$ and $(0,3)$. These sum to $0.1+0.02+0.02+0.05+0.09+0.075=0.355$

Overall, we have $P(X=m)=\begin{cases} 0.355 & m=0 \\ 0.335 & m=1 \\ 0.31 & m=-1 \\ 0 & \text{otherwise} \end{cases}$

ii) $P(Y=n)=\begin{cases} 0.1 & n=0 \\ 0.08 & n=-1 \\ 0.13 & n=1 \\ 0.4 & n=2 \\ 0.215 & n=-2 \\ 0.075 & n=3 \\ 0 & \text{otherwise} \end{cases}$.

iii) $P(X=Y)=P((X,Y)=(0,0))+P((X,Y)=(-1,-1))+P((X,Y)=(1,1))$

$=0.1+0.1+0.05=0.25$.

We therefore have, that

$P((X,Y)=(0,0)\mid X=Y)=\dfrac{0.1}{0.25}=0.4$

$P((X,Y)=(1,1)\mid X=Y)=\dfrac{0.1}{0.25}=0.4$ and

$P((X,Y)=(-1,-1)\mid X=Y)=\dfrac{0.05}{0.25}=0.2.$

Together, this gives $P(X=m\mid X=Y)=\begin{cases} 0.4 & m=0 \\ 0.2 & m=-1 \\ 0.4 & m=1 \\ 0 & \text{otherwise} \end{cases}$..

iv) $P(X=-1)=0.31$ hence, for example,

$$P(Y=-1 \mid X=-1)=\frac{P((X,Y)=(-1,-1))}{0.31}$$

hence $P(Y=n \mid X=-1)=\begin{cases} \frac{5}{31} & n=-1 \\ \frac{1}{31} & n=1 \\ \frac{5}{31} & n=-2 \\ \frac{20}{31} & n=2 \\ 0 & \text{otherwise} \end{cases}$.

v) $P(X=m \mid Y=3)=\begin{cases} 1 & m=0 \\ 0 & \text{otherwise} \end{cases}$

2. i) $$f_X(x)=\int_x^3 f(x,y)dy=\begin{cases}\int_x^3 \frac{2xy+1}{18}dy & 1<x<3\\ 0 & \text{otherwise}\end{cases}.$$

$$=\begin{cases}\left[\frac{xy^2+y}{18}\right]_x^3 & 1<x<3\\ 0 & \text{otherwise}\end{cases}$$

$$f_X(x)=\begin{cases}\frac{3+8x-x^3}{18} & 1<x<3\\ 0 & \text{otherwise}\end{cases}.$$

ii) $$f_Y(y)=\int_1^y f(x,y)dx=\begin{cases}\int_1^y \frac{2xy+1}{18}dx & 1<y<3\\ 0 & \text{otherwise}\end{cases}$$

$$=\begin{cases}\left[\frac{x^2y+x}{18}\right]_1^y & 1<y<3\\ 0 & \text{otherwise}\end{cases}$$

$$f_Y(y)=\int_1^y f(x,y)dx=\begin{cases}\frac{y^3-1}{18} & 1<y<3\\ 0 & \text{otherwise}\end{cases}.$$

iii)

To calculate $P(y-x>1)$, we can either evaluate

$\int_1^2\int_{1+x}^3 f(x,y)dydx$ or $\int_2^3\int_1^{y-1} f(x,y)dxdy$. These two integrals are equivalent.

$$\int_2^3\int_1^{y-1} f(x,y)dxdy=\int_2^3\left[\frac{x^2y+x}{18}\right]_1^{y-1}dy=\int_2^3\left[\frac{y^3-2y^2+y-2}{18}\right]dy$$

$$=\left[\frac{1}{18}\right]\left[\frac{y^4}{4}-\frac{2y^3}{3}+\frac{y^2}{2}-2y\right]_2^3=\left[\frac{1}{18}\right]\left[\frac{49}{12}\right]=\left[\frac{49}{216}\right].$$