University of Technology Sydney
School of Mathematical and Physical Sciences

Mathematical Statistics (37262) –
Tutorial 4

1. Let X and Y be independent standard normal random variables.
   Consider the variables defined as $S = X + Y$ and $D = X - Y$.
   i) Find statements for X and Y in terms of S and D.
   ii) Hence show that $|\det \boldsymbol{J}| = \frac{1}{2}$ where $\boldsymbol{J}$ as the Jacobian associated with this change of variables.
   iii) Calculate the joint distribution of S and D and hence show that S and D are independent and identically distributed. State the distributions of S and D.

2. The results of university subjects are graded as one of the following: High Distinction, Distinction, Credit, Pass or Fail.
   The final grades of three subjects are examined and the number of each grad awarded in each subject is given below.

	High Distinction	**Distinction**	**Credit**	**Pass**	**Fail**
Subject A	11	18	23	30	18
Subject B	6	7	11	18	8
Subject C	3	15	6	22	4

   i) Under the null hypothesis that the grade distribution is not dependent on the subject taken, calculate the table of expected grades awarded.
   ii) Calculate the Pearson statistic for this dataset and null hypothesis.
   iii) Perform a chi-squared goodness of fit test and assess whether you have reason to reject the belief that the grade distribution and the subject taken are independent. Perform this test with significance level 0.05.

3. Suppose that we have an observation $X = m$ of the variable $X \sim Bin(N, p)$ where N is a known positive integer and p is a probability parameter with a hypothesised value.

Under the central limit theorem, we have that $\dfrac{m - Np}{\sqrt{Np(1-p)}} \sim N(0,1)$.

Show that $\left[\dfrac{m - Np}{\sqrt{Np(1-p)}}\right]^2 = \dfrac{(m-Np)^2}{Np} + \dfrac{(N-m-N(1-p))^2}{N(1-p)}$.