University of Technology Sydney
School of Mathematical and Physical Sciences

Mathematical Statistics (37262) –
Tutorial 5

1.   Below are ten independent realisations of a *U[0,1]* random variable:

0.658, 0.646, 0.340, 0.165, 0.613, 0.376, 0.633, 0.933, 0.442, 0.289

a)   i)   Calculate the exact value of $\int_0^1 \sin(\pi x)dx$.

ii)   Using the ten random values above, employ the Monte Carlo Method to estimate the value the integral in part i).

b)   i)   Show that the exact value of $\int_0^\infty e^{-2x}\sin(x)dx$ is 0.2.

ii)   Using the ten random values above, employ the Monte Carlo Method to estimate the value the integral in part i).

Hint: For b) i) you will need to integrate by parts twice and establish a recurrence relationship.

2.   Let *X* be uniformly distributed, $X \sim U[0,1]$

i)   Write down $E(X)$ .Justify your answer.

ii)   Given that $Var(X) = \dfrac{1}{12}$, calculate <u>exactly</u> the probability that a realisation *x* of *X* lies within 3 standard deviations of the mean of the distribution.

That is, find $P\big(|x - E(X)| \le 3s\big)$ where s is the standard deviation of the distribution.

iii)   Show that the above probability is within the bounds given by Chebyshev's Inequality.