University of Technology Sydney
School of Mathematical and Physical Sciences

Mathematical Statistics (37262) –
Tutorial 5
SOLUTIONS

1.

a) i) $$\int_0^1 \sin(\pi x)dx = \left[\frac{-\cos(\pi x)}{\pi}\right]_0^1 = \frac{\cos(0)-\cos(\pi)}{\pi} = \frac{2}{\pi}.$$

ii)

$$\int_0^1 \sin(\pi x)dx \approx \frac{1}{10}\left[\sin(u_1\pi)+...+\sin(u_{10}\pi)\right]$$

$$\approx \frac{1}{10}\left[0.87932+...+0.78823\right] \approx 0.7905$$

b) i) Integration by parts once gives

$$\int_0^\infty e^{-2x}\sin(x)dx = \left[-e^{-2x}\cos(x)\right]_0^\infty - \int_0^\infty 2e^{-2x}\cos(x)dx = 1-\int_0^\infty 2e^{-2x}\cos(x)dx$$

Integrating by parts again gives

$$\begin{aligned}\int_0^\infty e^{-2x}\sin(x)dx &= 1-\int_0^\infty 2e^{-2x}\cos(x)dx \\ &= 1-\left(\left[-2e^{-2x}\sin(x)\right]_0^\infty - \int_0^\infty 4e^{-2x}\sin(x)dx\right) \\ &= 1-4\int_0^\infty e^{-2x}\sin(x)dx\end{aligned}$$

We now know that $\int_0^\infty e^{-2x}\sin(x)dx = 1-4\int_0^\infty e^{-2x}\sin(x)dx$ hence

$5\int_0^\infty e^{-2x}\sin(x)dx = 1$ so $\int_0^\infty e^{-2x}\sin(x)dx = 0.2$.

ii) The change of variable $y=1-e^{-x}$ gives $x=-\ln(1-y)$ and hence $\frac{dy}{dx}=1-y$

$$\int_0^\infty e^{-2x}\sin(x)dx \approx \frac{1}{10}\left[\frac{e^{-2(-\ln(1-u_1))}\sin(-\ln(1-u_1))}{1-u_1}+...+\frac{e^{-2(-\ln(1-u_{10}))}\sin(-\ln(1-u_{10}))}{1-u_{10}}\right]$$

$$\approx \frac{1}{10}\left[0.30048+...+0.23784\right] \approx 0.2503$$

2. i) $E(X)=\int_0^1 x\,dx=\left[\frac{x^2}{2}\right]_0^1=0.5$.

(Alternatively, you could reason that as it is uniformly distributed between 0 and 1, then the expectation has to be the midpoint i.e. 0.5)

ii) $P\left(0.5-\frac{3}{\sqrt{12}}\le X\le 0.5+\frac{3}{\sqrt{12}}\right)=\int_{0.5-\frac{3}{\sqrt{12}}}^{0.5+\frac{3}{\sqrt{12}}} f(x)\,dx=1$ (since $0.5+\frac{3}{\sqrt{12}}>1$)

iii) The bounds from Chebyshev's Inequality tell us that

$P\left(0.5-\frac{3}{\sqrt{12}}\le X\le 0.5+\frac{3}{\sqrt{12}}\right)\ge 1-\frac{1}{3^2}=\frac{8}{9}$ which is less than the true value from part ii).