Question 4. (10 marks)

Important: marks will only be awarded for fully worked solutions, showing all steps.

(a)   The general solution to Laplace's equation in 2D polar coordinates is

$$u(r, \theta) = A_0 + B_0 \ln r + \sum_{m=-\infty}^{\infty} \left[a_m r^{|m|} + b_m r^{-|m|} \right] e^{im\theta} \ .$$

Use this to solve
$$\nabla^2 u = 0$$
on the 2D circular domain $r \leq \frac{1}{2}$, with boundary condition

$$u(\frac{1}{2}, \theta) = \sin(2\theta) + \frac{1}{2} \cos(2\theta)$$

at $r = \frac{1}{2}$. Make sure that you state all your reasoning.

(7 marks)

(b)   Check you answer from part (a) by showing that it obeys the partial differential equation.
(3 marks)