
Question 4. (10 marks)

Important: marks will only be awarded for fully worked solutions, showing all steps.

- (a) By separating variables in two dimensional polar coordinates, find the general solution for $u(r, \theta)$ if u obeys the Helmholtz equation

$$\nabla^2 u + k^2 u = 0$$

on the domain

$$r \leq 4,$$

subject to the restriction that u remains finite on this domain. (5 marks)

- (b) Write the general solution if u does not depend on θ . (1 marks)

- (c) Find the two smallest positive values of k given that

$$u(4, \theta) = 0.$$

The first two zeros of the Bessel function of the first kind are

$$\begin{aligned} j_{0,1} &= 2.4048 \\ j_{0,2} &= 5.5201 . \end{aligned}$$

(2 marks)

- (d) How many possible values of k are there for the boundary conditions given in part (c)? Give a brief justification for your answer. (2 marks)
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