Question 3. (10 marks)

Important: marks will only be awarded for fully worked solutions, showing all steps.

A scalar function $V(x, z)$ obeys Laplace's equation

$$\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial z^2} = 0$$

in the region

$$(0 \leq x \leq 3), \quad (0 \leq z \leq 1),$$

and is subject to the boundary conditions

$$V(x, 0) = V(x, 1) = V(0, z) = 0 \ ,$$
$$V(3, z) = \sin(2\pi z) \ .$$

Use separation of variables to compute $V(x, z)$.