

3)

$$V(x, z): V_{xx} + V_{zz} = 0$$

$$x \in [0, 3]$$

$$V(x, 0) = V(x, 1) = V(0, z) = 0$$

$$z \in [0, 1]$$

$$V(3, z) = \sin(2\pi z)$$

$$\text{Let } V(x, z) = X(x) Z(z)$$

$$\therefore X''Z + Z''X = 0 \quad (\div XZ)$$

$$\frac{X''}{X} + \frac{Z''}{Z} = 0$$

$$\frac{X''}{X} = \lambda \quad \frac{Z''}{Z} = -\lambda \quad (\text{separate})$$

$$X'' = \lambda X$$

$$m^2 e^{mx} = \lambda e^{mx}$$

$$m^2 - \lambda = 0$$

$$\Rightarrow m = \pm \sqrt{\lambda}$$

Has two real solutions

therefore characteristic method gives

$$X(x) = \sinh(\sqrt{\lambda} x)$$

$$(V(0, z) = 0 \text{ implies sinh})$$

$$\therefore V(x, y) = \sum_{\substack{n=-\infty \\ n \neq 0}}^{\infty} c_n \sin(\sqrt{\lambda_n} z) \sinh(\sqrt{\lambda_n} x)$$

$$\sin(2\pi z) = \sum_{\substack{n=-\infty \\ n \neq 0}}^{\infty} c_n \sin(\pi n z) \sinh(\pi n z)$$

$$\sin(2\pi z) = c_2 \sin(\pi z) \sinh(6\pi)$$

(Equate coefficients with boundary)

$$\Rightarrow c_2 = \frac{1}{\sinh(6\pi)} \quad c_n = 0 \quad n \neq 2$$

$$\therefore V(x, y) = \frac{\sin(\pi z) \sinh(\pi x)}{\sinh(6\pi)}$$

(Fourier Projection may also be used to arrive at the same conclusion)