

4)

$$a) \nabla^2 f = \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial f}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2 f}{\partial \theta^2}$$

$$r \in [0, 4]$$

$$u < \infty \quad \forall (r, \theta)$$

$$\nabla^2 u + k^2 u = 0$$

$$\Rightarrow \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial u}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2 u}{\partial \theta^2} + k^2 u = 0$$

$$u(r, \theta) = R(r) \Theta(\theta)$$

$$\Theta \frac{1}{r} \frac{\partial}{\partial r} (r R') + \frac{1}{r^2} R \Theta'' + k^2 R \Theta = 0$$

$$(\div \Theta R, \times r^2)$$

$$\frac{r \frac{\partial}{\partial r} (r R')}{R} + \frac{\Theta''}{\Theta} + k^2 r^2 = 0$$

$$\frac{r (r R'' + R')}{R} + k^2 r^2 = \lambda$$

$$\frac{\Theta''}{\Theta} = -\lambda$$

characteristic equation solved in Q3!

$$r^2 R'' + r R' + k^2 r^2 R = \lambda R$$

$$\Theta(\theta) = e^{i\sqrt{\lambda}\theta}$$

Bessel Equation; solved by Bessel functions of first kind if bound by non-singularity

$$R(r) = J_{\sqrt{\lambda}}(kr)$$

$$\therefore u(r, \theta) = J_{\sqrt{\lambda}}(kr) e^{i\sqrt{\lambda}\theta}$$

$$b) u(r) = J_{\sqrt{\lambda}}(kr) \quad \text{if } u \text{ independent of } \theta$$

$$c) u(4, \theta) = 0 \Rightarrow J_{\sqrt{\lambda}}(k4) = 0 \Rightarrow K4 = j_{0,1} \Rightarrow K_1 = \frac{j_{0,1}}{4} \approx \underline{0.6012}$$

$$\text{or } = j_{0,2} \Rightarrow K_2 = \frac{j_{0,2}}{4} \approx \underline{1.38025}$$

d) Infinite, for each zero of  $J_{\sqrt{\lambda}}$  the following procedure can be carried out and since Bessel functions have infinitely many zeroes, there are infinite  $K$ .