

1)

a) $\vec{F} = \begin{pmatrix} y(z+1) \\ -yz \\ xz \end{pmatrix}$

$S = \{(x, y, z): (x-1)^2 + y^2 + z^2 = 9, z \geq 0\}$
(upward oriented)

(Stokes)
 $\iint_S \nabla \times \vec{F} \cdot d\vec{S} = \oint_{\partial S} \vec{F} \cdot d\vec{r}$

$(x-1)^2 + y^2 = 9$
 $\vec{r}(t) = \begin{pmatrix} 1+3\cos t \\ 3\sin t \\ 0 \end{pmatrix} \quad t \in [0, 2\pi]$

$= \int_0^{2\pi} \begin{pmatrix} 3\sin t \\ 0 \\ 0 \end{pmatrix} \cdot \begin{pmatrix} -3\sin t \\ 3\cos t \\ 0 \end{pmatrix} dt$

$\vec{r}'(t) = \begin{pmatrix} -3\sin t \\ 3\cos t \\ 0 \end{pmatrix}$

$= \int_0^{2\pi} -9 \sin^2 t \, dt$

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$= -9 \left[\frac{t}{2} + \frac{\sin(2t)}{4} \right]_0^{2\pi} = -9 \left[\frac{2\pi}{2} - 0 \right] = \underline{\underline{-\frac{9}{2}\pi}}$

b) The boundary of this circular disk is the same (as) therefore the parametrisation is identical and Stokes's theorem may be reapplied. Since orientation is also the same, the integral has the same sign too, hence:

$\iint_{S_2} \nabla \times \vec{F} \cdot d\vec{S} = \underline{\underline{-\frac{9}{2}\pi}}$