Problem Set 0 - solutions

1. a) $\underset{\sim}{a} + \underset{\sim}{b} = (1,2,2) + (2,-3,1)$

$= (2+1,\ 2+(-3),\ 2+1)$

$= (3,-1,3)$ vector

b) $2\underset{\sim}{a} - \underset{\sim}{b} = 2(1,2,2) - (2,-3,1)$

$= (2,4,4) - (2,-3,1)$

$= (0,7,3)$ vector

c) $|\underset{\sim}{a}| = \sqrt{1^2+2^2+2^2} = \sqrt{9} = 3$ scalar

d) $\underset{\sim}{a}\cdot\underset{\sim}{b} = a_1b_1 + a_2b_2 + a_3b_3$

$= 2 - 6 + 2$

$= -2$ scalar

2. a) $\underset{\sim}{a}\cdot\underset{\sim}{b} = 2+0+0 = 2$

b) We know

$\underset{\sim}{a}\cdot\underset{\sim}{b} = |\underset{\sim}{a}||\underset{\sim}{b}|\cos\theta$

so $\cos\theta = \dfrac{\underset{\sim}{a}\cdot\underset{\sim}{b}}{|\underset{\sim}{a}||\underset{\sim}{b}|} = \dfrac{2}{\sqrt{2^2+0+1^2}\sqrt{1^2+(-3)^2+0}} = \dfrac{2}{\sqrt{5}\sqrt{10}}$

$= \dfrac{2}{\sqrt{50}} \Rightarrow \theta = 1.28$

c) $\hat{\underset{\sim}{b}} = \dfrac{\underset{\sim}{b}}{|\underset{\sim}{b}|} = \dfrac{(1,-3,0)}{\sqrt{1^2+(-3)^2+0}} = \dfrac{1}{\sqrt{10}}(1,-3,0)$

d) The vector parallel to $\underset{\sim}{b}$ and twice as long is

$2\underset{\sim}{b} = (2,-6,0)$

e) The scalar projection is

$$\underset{\sim}{a} \cdot \hat{\underset{\sim}{b}} = (2,0,1) \cdot \hat{\underset{\sim}{b}}$$

$$= (2,0,1) \cdot \frac{(1,-3,0)}{\sqrt{10}}$$

$$= \frac{2+0+0}{\sqrt{10}} = \frac{2}{\sqrt{10}}$$

3. a)

$$\underset{\sim}{a} \times \underset{\sim}{b} = \begin{vmatrix} \hat{\underset{\sim}{i}} & \hat{\underset{\sim}{j}} & \hat{\underset{\sim}{k}} \\ 1 & -1 & 3 \\ 3 & 1 & -1 \end{vmatrix} = \hat{\underset{\sim}{i}}(1-3) - \hat{\underset{\sim}{j}}(-1-9) + \hat{\underset{\sim}{k}}(1+3)$$

$$= -2\hat{\underset{\sim}{i}} + 10\hat{\underset{\sim}{j}} + 4\hat{\underset{\sim}{k}} \quad \left(= (-2,10,4) \right)$$

b)

$$\underset{\sim}{a} \times \underset{\sim}{b} = \begin{vmatrix} \hat{\underset{\sim}{i}} & \hat{\underset{\sim}{j}} & \hat{\underset{\sim}{k}} \\ 2 & 4 & -3 \\ 2 & -3 & 4 \end{vmatrix} = \hat{\underset{\sim}{i}}(16-9) - \hat{\underset{\sim}{j}}(8+6) + \hat{\underset{\sim}{k}}(-6-8)$$

$$= 7\hat{\underset{\sim}{i}} - 14\hat{\underset{\sim}{j}} - 14\hat{\underset{\sim}{k}}$$

c)

$$\underset{\sim}{a} \times \underset{\sim}{b} = \begin{vmatrix} \hat{\underset{\sim}{i}} & \hat{\underset{\sim}{j}} & \hat{\underset{\sim}{k}} \\ 1 & 1 & 0 \\ 1 & -1 & 0 \end{vmatrix}$$

$$= \hat{\underset{\sim}{i}}(0-0) - \hat{\underset{\sim}{j}}(0-0) + \hat{\underset{\sim}{k}}(-1-1)$$

$$= -2\hat{\underset{\sim}{k}}$$

4. a)

$$\frac{\partial f}{\partial x} = 4x - 3y$$

$$\frac{\partial f}{\partial y} = -3x + 8y$$

b)

$$\frac{\partial g}{\partial x} = 2\cos(2x+3y)$$

$$\frac{\partial g}{\partial y} = 3\cos(2x+3y)$$

c)

$$\frac{\partial f}{\partial x} = yz\cos(xyz)$$

$$\frac{\partial f}{\partial y} = xz\cos(xyz)$$

$$\frac{\partial f}{\partial z} = 2z + xy\cos(xyz)$$

d)

$$\frac{\partial f}{\partial y} = \frac{\partial}{\partial y}(yz^{-1}) = z^{-1} = \frac{1}{z}$$

$$\frac{\partial f}{\partial z} = \frac{\partial}{\partial z}(yz^{-1}) = y\left(-\frac{1}{z^2}\right) = -\frac{y}{z^2}$$

5. We have

$$f(x,y) = x^2 + 3xy + y^2$$

We want to find f_{xx}, f_{yy}, f_{xy} and f_{yy}.

Now

$$\frac{\partial f}{\partial x} = 2x + 3y \qquad \frac{\partial f}{\partial y} = 3x + 2y$$

$$\Rightarrow \frac{\partial^2 f}{\partial x^2} = 2 \qquad \frac{\partial^2 f}{\partial y^2} = 2$$

$$\frac{\partial^2 f}{\partial x \partial y} = 3 \qquad \frac{\partial^2 f}{\partial y \partial x} = 3$$

So

$$\underset{f_{xy}}{\frac{\partial^2 f}{\partial x \partial y}} = \underset{f_{yx}}{\frac{\partial^2 f}{\partial y \partial x}}$$ as required.

(In this case we <u>also</u> have $f_{xx} = f_{yy}$ but this is a coincidence!)