

Vector Calculus and PDEs 37336
Problem Set 0 (for preparation and practice)

These questions are intended as revision of material covered in Maths 1/Maths 2- you have to make sure that you know vectors and partial derivatives before moving on. If you're struggling (or need a refresher) you can read through Chapters 1-2 in the Course Notes, and try the questions at the end.

1. Let $\mathbf{a} = (1, 2, 2)$ and $\mathbf{b} = (2, -3, 1)$. Calculate:

- a) $\mathbf{a} + \mathbf{b}$
- b) $2\mathbf{a} - \mathbf{b}$
- c) $|\mathbf{a}|$
- d) $\mathbf{a} \cdot \mathbf{b}$

State whether each quantity is a vector or a scalar.

2. Let $\mathbf{a} = (2, 0, 1)$ and $\mathbf{b} = (1, -3, 0)$. Calculate:

- a) $\mathbf{a} \cdot \mathbf{b}$
- b) The angle θ between the two vectors
- c) $\hat{\mathbf{b}}$
- d) A vector parallel to $\mathbf{b}$ and twice as long
- e) The scalar projection of the vector $\mathbf{a}$ in the direction of $\mathbf{b}$

3. Find the cross product $\mathbf{a} \times \mathbf{b}$ for the following vectors $\mathbf{a}$ and $\mathbf{b}$:

- a) $\mathbf{a} = \hat{\mathbf{i}} - \hat{\mathbf{j}} + 3\hat{\mathbf{k}}$, $\mathbf{b} = 3\hat{\mathbf{i}} + \hat{\mathbf{j}} - \hat{\mathbf{k}}$
- b) $\mathbf{a} = 2\hat{\mathbf{i}} + 4\hat{\mathbf{j}} - 3\hat{\mathbf{k}}$, $\mathbf{b} = 2\hat{\mathbf{i}} - 3\hat{\mathbf{j}} + 4\hat{\mathbf{k}}$
- c) $\mathbf{a} = \hat{\mathbf{i}} + \hat{\mathbf{j}}$, $\mathbf{b} = \hat{\mathbf{i}} - \hat{\mathbf{j}}$

4. Evaluate *all* first partial derivatives of the following functions:

- a) $f(x, y) = 2x^2 - 3xy + 4y^2$
- b) $g(x, y) = \sin(2x + 3y)$
- c) $f(x, y, z) = z^2 + \sin(xyz)$
- d) $f(y, z) = y/z$

5. Evaluate *all* second partial derivatives of

$$f(x, y) = x^2 + 3xy + y^2 ,$$

and verify that $f_{xy} = f_{yx}$.