1. a) A vector parallel to the line joining $\underset{\sim}{a} = \langle 1, 1, 0 \rangle$ and $\underset{\sim}{b} = \langle 0, 1, 1 \rangle$ is

$$\underset{\sim}{p} = \underset{\sim}{b} - \underset{\sim}{a}$$
$$= \langle -1, 0, 1 \rangle$$

b) A parametric representation of the path is

$$\underset{\sim}{r}(t) = \underset{\sim}{p}t + \underset{\sim}{a}$$
$$= \langle -1, 0, 1 \rangle t + \langle 1, 1, 0 \rangle$$
$$= \langle 1-t, 1, t \rangle$$
$$= (1-t)\hat{\underset{\sim}{i}} + \hat{\underset{\sim}{j}} + t\hat{\underset{\sim}{k}} .$$

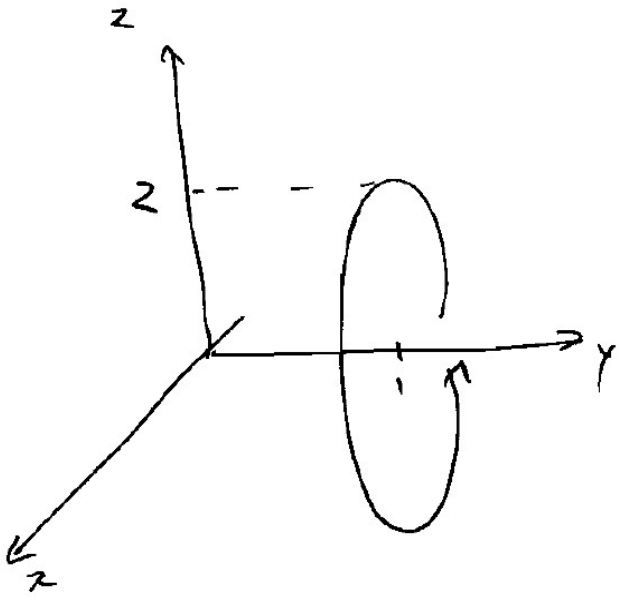
2. The infinitesimal arc-length is

$$ds = \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2 + \left(\frac{dz}{dt}\right)^2}\, dt$$
$$= \sqrt{1^2 + (2t)^2 + 0^2}\, dt$$
$$= \sqrt{1 + 4t^2}\, dt .$$

The integral is therefore

$$\int_C \sqrt{z + 4x^2}\, ds = \int_0^1 \sqrt{1 + 4t^2} \cdot \sqrt{1 + 4t^2}\, dt$$
$$= \int_0^1 (1 + 4t^2)\, dt = \left[t + \frac{4}{3}t^3\right]_0^1 = \frac{7}{3} .$$

3.



A parametric representation is

$$\underset{\sim}{r}(t) = x(t)\,\underset{\sim}{\hat{i}} + y(t)\,\underset{\sim}{\hat{j}} + z(t)\,\underset{\sim}{\hat{k}}$$

where

$$x(t) = 2\cos t$$

$$y(t) = 1$$

$$z(t) = 2\sin t$$

where $0 \le t \le 2\pi$.

4.

$$\int_C \underline{F}\cdot d\underline{r} = \int_0^2 \langle yz, xz, xy\rangle \cdot \left(\frac{d\underline{r}}{dt}\right) dt$$

$$= \int_0^2 \langle yz, xz, xy\rangle \cdot \langle -1, 1, 2\rangle \, dt$$

$$= \int_0^2 (-yz + xz + 2xy)\, dt$$

remember these are all functions of t!

$$= \int_0^2 \left[-(t+1)2t + (2-t)2t + 2(2-t)(t+1)\right] dt$$

$$= \int_0^2 \left[-2t^2 - 2t + 4t - 2t^2 + 2(2t + 2 - t^2 - t)\right] dt$$

$$= \int_0^2 \left[-4t^2 + 2t + 4t + 4 - 2t^2 - 2t\right] dt$$

$$= \int_0^2 \left[-6t^2 + 4t + 4\right] dt$$

$$= \left[-2t^3 + 2t^2 + 4t\right]_0^2$$

$$= -2\times 8 + 2\times 4 + 4\times 2$$

$$= 0$$

5. From the fundamental theorem in ~~ϕ~~ ϕ,

$$\int_C \underset{\sim}{F} \cdot d\underset{\sim}{r} = \int_C \nabla\phi \cdot d\underset{\sim}{r}$$

$$= \phi(\underset{\sim}{r}_1) - \phi(\underset{\sim}{r}_0)$$

where $\underset{\sim}{r}_1 = \langle 2, \frac{\pi}{2} \rangle$ $\quad \underset{\sim}{r}_0 = \langle 1, 0 \rangle$

so

$$\int_C \underset{\sim}{F} \cdot d\underset{\sim}{r} = \phi(2, \tfrac{\pi}{2}) - \phi(1, 0)$$

$$= \left(2^2 \times \frac{\pi}{2} + 2\sin\frac{\pi}{2}\right) - \left(1^2 \times 0 + 1 \times \sin 0\right)$$

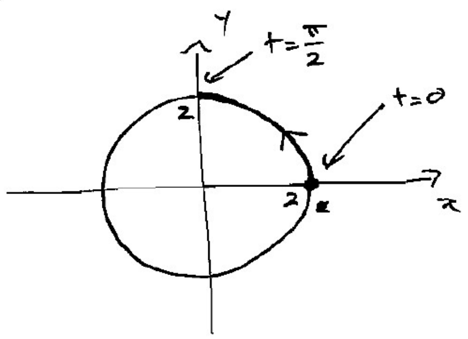
$$= 2\pi + 2 .$$

6. A suitable parameterisation of the curve C is

$$x(t) = 2\cos t$$

$$y(t) = 2\sin t$$

$$z(t) = 1$$



where $0 \le t \le \frac{\pi}{2}$.

The line integral is

$$\int_C \underset{\sim}{F} \cdot d\underset{\sim}{r} = \int_0^{\frac{\pi}{2}} \left(\underset{\sim}{F} \cdot \frac{d\underset{\sim}{r}}{dt}\right) dt$$

$$= \int_0^{\frac{\pi}{2}} \left(2\cos t \cdot 2\sin t\, \hat{\underset{\sim}{i}} + (2\cos t - 2\sin t)\hat{j} \right)$$

$$\cdot \left(-2\sin t\, \hat{\underset{\sim}{i}} + 2\cos t\, \hat{j} \right) dt$$

$$= \int_0^{\frac{\pi}{2}} \left(4\cos t \sin t \cdot (-2\sin t) \cancel{+ 2\cos t(\cos t} \right.$$

$$\left. + (2\cos t - 2\sin t) \cdot 2\cos t \right) dt$$

$$= \int_0^{\frac{\pi}{2}} \left(-8\sin^2 t \cos t + 4\cos^2 t - 4\cos t \sin t \right) dt$$

$$= \left[-\frac{8}{3}\sin^3 t \right]_0^{\frac{\pi}{2}} + \int_0^{\frac{\pi}{2}} \frac{4}{2}(1 + \cos 2t)\, dt + \left[-\frac{4}{2}\sin^2 t \right]_0^{\frac{\pi}{2}}$$

$$= -\frac{8}{3} + \left[2t + 2 \cdot \frac{1}{2}\sin 2t \right]_0^{\frac{\pi}{2}} - 2$$

$$= -\frac{8}{3} + 2\frac{\pi}{2} + \cancel{\sin \pi} - 2 = -\frac{8}{3} - 2 + \pi$$

$$= -\frac{14}{3} + \pi.$$

7. $\underset{\sim}{F}$ is conservative iff $\nabla \times \underset{\sim}{F} = \underset{\sim}{0}$.

$$\nabla \times \underset{\sim}{F} = \begin{vmatrix} \hat{\underset{\sim}{i}} & \hat{\underset{\sim}{j}} & \hat{\underset{\sim}{k}} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ x^3+4xy & y^3+4xy & 0 \end{vmatrix}$$

$$= \hat{\underset{\sim}{i}}(0-0) - \hat{\underset{\sim}{j}}(0-0) + \hat{\underset{\sim}{k}}(4y-4z)$$

$$= \hat{\underset{\sim}{k}}(4y-4z)$$

In this case $\nabla \times \underset{\sim}{F} \neq 0$ so $\underset{\sim}{F}$ is not conservative.

8. First check that $\nabla \times \underset{\sim}{F} = 0$ (If not, then we can't find a potential)

$$\nabla \times \underset{\sim}{F} = \begin{vmatrix} \hat{\underset{\sim}{i}} & \hat{\underset{\sim}{j}} & \hat{\underset{\sim}{k}} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ \sin y & x\cos y & \sin z \end{vmatrix}$$

$$= \underset{\sim}{\hat{i}}(0-0) - \underset{\sim}{\hat{j}}(0-0) + \underset{\sim}{\hat{k}}(\cos y - \cos y)$$

$$= \underset{\sim}{0}$$

So $\underset{\sim}{F}$ is conservative.

The potential is a function ϕ such that

$$\underset{\sim}{F} = \nabla\phi$$

i.e.

$$\underline{\frac{\partial\phi}{\partial x} = \sin y}, \quad \frac{\partial\phi}{\partial y} = x\cos y, \quad \frac{\partial\phi}{\partial z} = -\sin z$$

Integrate the 1st equation wrt x:

$$\phi(x,y,z) = \int \sin y \, dx + A(y,z)$$

$$= x\sin y + A(y,z)$$

So $$\frac{\partial\phi}{\partial y} = x\cos y + \frac{\partial A}{\partial y} \Rightarrow x\cos y + \frac{\partial A}{\partial y} = x\cos y$$

So $$\frac{\partial A}{\partial y} = 0.$$

Integrate wrt y:

$$A(y,z) = B(z)$$

So

$$\phi(x,y,z) = x\sin y + B(z)$$

$$\Rightarrow \frac{\partial \phi}{\partial z} = \frac{\partial B}{\partial z} \Rightarrow \frac{\partial B}{\partial z} = -\sin z$$

integrating wrt z:

$$B(z) = \cos z + C$$