Vector Calculus and PDEs 37336
Problem Set 5: Flux and surface integrals

1. Find a vector normal to the surface $z = x + y^2$ at the point $(1, 2, 5)$.

2. For the surface with parametric equations $x = uv$, $y = u + v$, $z = u - v$, find expressions for (a) the infinitessimal vector surface element $d\mathbf{S}$, and (b) the infinitessimal area dS.

3. Evaluate the surface integral
$$\iint_S yz\ dS$$
where S is surface represented by the equation $x + 2y + z = 1$, with $x \geq 0$, $y \geq 0$, $z \geq 0$.

4. * Find the area of the surface represented parametrically by
$$\mathbf{r}(u, v) = u \cos v\hat{\mathbf{i}} + u \sin v\hat{\mathbf{j}} + v\hat{\mathbf{k}}$$
with $0 \leq u \leq 1$, $0 \leq v \leq \pi$.
Note: This one has a pretty hard integral at the end - you might want to leave the area in integral form and come back to it when you have time.

5. Calculate the surface integral of the vector function
$$\iint_S \mathbf{F} \cdot \mathbf{dS}$$
where $\mathbf{F} = x\hat{\mathbf{i}} - z^2\hat{\mathbf{k}}$ and S is the surface defined by the equations
$$y + z = 2 \quad , 0 \leq x \leq 2, \quad 0 \leq y \leq 2 \quad .$$

6. Calculate the surface integral $\iint_S \mathbf{F} \cdot d\mathbf{S}$ of the vector function
$$\mathbf{F}(\rho, \varphi, \theta) = \frac{k}{\rho^2}\hat{\rho}$$
where S is a hemisphere centred at the origin, with radius 4 and existing in the positive half-volume $x > 0$.