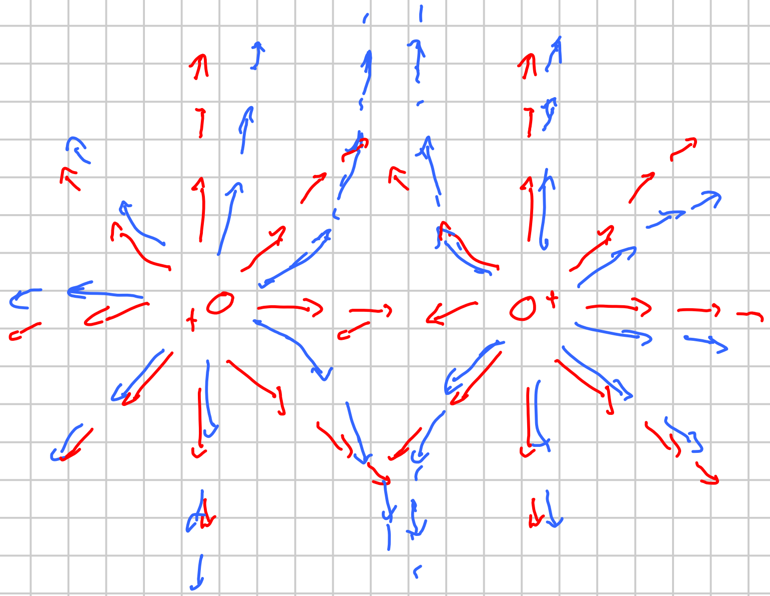
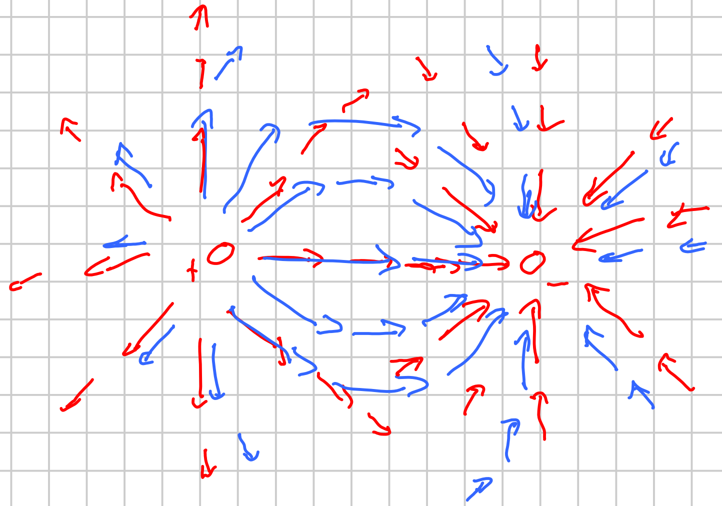
1. a)



b)



2. From Gauss's Law

$$\iint_S \underset{\sim}{E} \cdot d\underset{\sim}{S} = \frac{Q_{enc}}{\varepsilon_0}$$

enclosed charge.

$$= \frac{3}{\varepsilon_0}$$

3. From Gauss's Law in differential form

$$\nabla \cdot \underset{\sim}{E} = \frac{\rho(\underset{\sim}{r})}{\varepsilon_0}$$

so

$$\rho(\underset{\sim}{r}) = \varepsilon_0 \nabla \cdot \underset{\sim}{E}$$

$$= \varepsilon_0 \nabla \cdot \left(k r^3 \hat{\underset{\sim}{r}} \right)$$

In spherical coordinates,

$$\nabla \cdot \underset{\sim}{F} = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 F_r\right) + \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}\left(F_\theta \sin\theta\right) + \frac{1}{r\sin\theta}\frac{\partial F_\varphi}{\partial \varphi}$$

Here

$$\nabla \cdot \left(k r^3 \hat{r}\right) = \frac{1}{r^2}\frac{\partial}{\partial r}\left(k r^4\right)$$

$$= \frac{1}{r^2} \cdot 4 k r^3$$

$$= 4 k r.$$

So

$$\rho(r) = \varepsilon_0 \nabla \cdot \underset{\sim}{E}$$

$$= \varepsilon_0 4 k r.$$

4. We want to find V such that

$$-\nabla V = \underset{\sim}{E}$$

Or

$$\nabla V = -\underset{\sim}{E}$$

$$= -\left(Cx + D\right)\hat{i}$$

Now

$$\frac{\partial V}{\partial x} = -Cx + D$$

So

$$V(x,y,z) = -\frac{1}{2}Cx^2 + Dx + g(y,z)$$

We also have

$$\frac{\partial V}{\partial y} = 0$$

So

$$\frac{\partial g}{\partial y} = 0 \implies g = p(z) + q$$

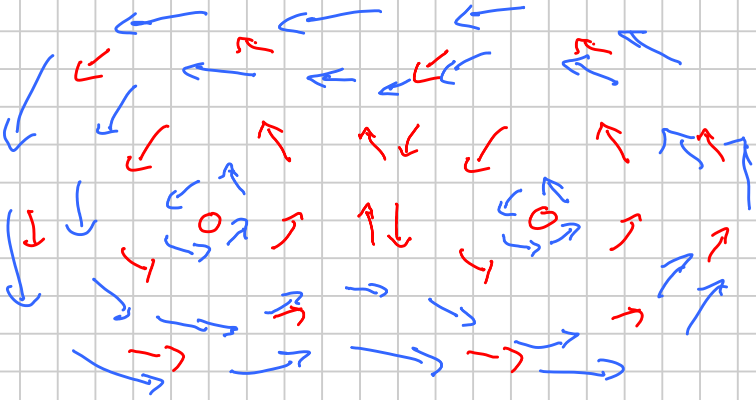
Finally

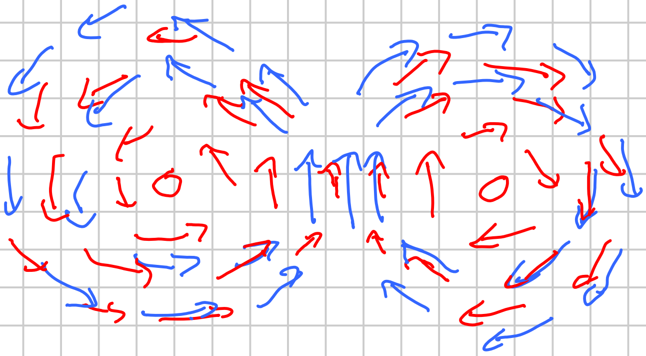
$$\frac{\partial V}{\partial z} = 0 \implies p(z) = \text{constant}$$

So

$$V = -\frac{1}{2}Cx^2 + Dx + \text{constant}.$$

5. a)





6. Now

$$\nabla^2 \underset{\sim}{E} = \nabla^2 A\cos\left(\frac{\omega}{c}x - \omega t\right)\hat{j}$$

$$= \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\right) A\cos\left(\frac{\omega}{c}x - \omega t\right)\hat{j}$$

$$= \frac{-\omega^2}{c^2} A\cos\left(\frac{\omega}{c}x - \omega t\right)\hat{j} .$$

$$= \frac{-\omega^2}{c^2}\underset{\sim}{E}$$

Also

$$\frac{1}{c^2}\frac{\partial^2}{\partial t^2}\underset{\sim}{E} = \frac{1}{c^2}\frac{\partial^2}{\partial t^2} A\cos\left(\frac{\omega}{c}x - \omega t\right)\hat{j}$$

$$= \frac{1}{c^2}\left(-\omega^2\right) A\cos\left(\frac{\omega}{c} - \omega t\right)\hat{j} .$$

$$= \frac{-\omega^2}{c^2} A\cos\left(\frac{\omega}{c} - \omega t\right)\hat{j} = \frac{-\omega^2}{c^2}\underset{\sim}{E} .$$

Therefore $\nabla^2 \underset{\sim}{E} = \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\underset{\sim}{E}$ as required.

7. We have, re-arranging

$$\underset{\sim}{B} = \frac{1}{i\omega} \nabla \times \underset{\sim}{E}$$

$$= \frac{1}{i\omega} \begin{vmatrix} \hat{\underset{\sim}{i}} & \hat{\underset{\sim}{j}} & \hat{\underset{\sim}{k}} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 0 & A\cos\left(\frac{\omega}{c}x - \omega t\right) & 0 \end{vmatrix}$$

$$= \frac{1}{i\omega}\left[\hat{\underset{\sim}{i}}\,(0-0) - \hat{\underset{\sim}{j}}(0-0) + \hat{\underset{\sim}{k}}\left(-A\frac{\omega}{c}\sin\left(\frac{\omega}{c}x - \omega t\right)\right)\right]$$

$$= -\frac{A}{ic}\sin\left(\frac{\omega}{c}x - \omega t\right)\hat{\underset{\sim}{k}} .$$

So $\underset{\sim}{B}$ points in the z-direction, so it is always perpendicular to $\underset{\sim}{E}$, which points in the y-direction.