Advanced Calculus Class Test
Time Allowed: 50 minutes.

Advanced Calculus. Alternative Class Test
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(1) Obtain all solutions of the equation $z^5 + 1 = 0$.

(2) Prove that the function $f(z) = \cosh z$ is differentiable. Hint: Put $z = x + iy$ and use the Cauchy- Riemann equations.

(3) Obtain a Laurent series expansion for the function

$$f(z) = \frac{\sin(tz^2)}{z^7}$$

about the point $z = 0$. What is the order of the pole at $z = 0$? What is the value of the residue at the pole?

(4) Let C be the circle of radius 1 centered at $z = 2$. Obtain the value of the integral

$$\int_C \frac{e^{\frac{1}{2}\pi iz}}{z^2 - 4} dz.$$

(5) Use Green's Theorem to evaluate the contour integral

$$\oint_C (y^3 + x^2)dx + (x^3 + y^2)dy$$

in which C is the boundary of the circle $x^2 + y^2 = 4$, traversed counterclockwise.

(6) Use the substitution $z = e^{i\theta}$ to evaluate the integral

$$\int_0^{2\pi} \frac{3}{4 + \sin\theta} d\theta.$$

Formulas Over Page

Useful information

$$
\begin{aligned}
\frac{1}{1-z} &= 1 + z + z^2 + z^3 + \cdots, \ |z| < 1. \\
e^z &= 1 + z + \frac{z^2}{2!} + \frac{z^3}{3!} + \cdots \\
\sin z &= z - \frac{z^3}{3!} + \frac{z^5}{5!} - \cdots \\
e^{iz} &= \cos z + i \sin z. \\
\int_\gamma f(z) dz &= \int_a^b f(\gamma(t))\gamma'(t) dt, \ \text{where } \gamma = \gamma(t), \ t \in [a, b], \\
f^{(n)}(z) &= \frac{n!}{2\pi i} \int_\gamma \frac{f(\xi)}{(\xi - z)^{n+1}} d\xi.
\end{aligned}
$$