Advanced Calculus. Class Test 3, Thursday.

Non Programmable Calculators may be used.

(1) Let

$$f(z) = \frac{1}{z^3} e^{2tz}.$$

Obtain a Laurent series expansion for f about $z = 0$. Determine the location and order of the pole and the value of the residue at the pole.

(2) Obtain a power series expansion for the function

$$f(z) = \frac{z}{1 - z^2}, \text{valid for} |z| < 1.$$

Obtain an expansion in powers of $1/z$, valid for $|z| > 1$. Determine the poles of f and the values of the residues at the poles.

(3) Evaluate the integral

$$\int_0^{2\pi} \frac{2}{3 - \sin t} dt,$$

by making the substitution $z = e^{it},\ t \in [0, 2\pi]$.

(4) Use residues to evaluate the integral

$$\int_0^{\infty} \frac{x^2}{x^4 + 1} dx.$$