Advanced Calculus 37234: Week 1 Preparation Work

These are some questions to consider before formal lectures begin. They will be discussed in detail over the course of the subject.

(i) What is the purpose of calculus? Why was it invented and what is it for?

(ii) What is a continuous function? How does the requirement that a function be continuous constrain the way a function behaves?

(iii) Review the definition of the derivative of a function $f : \mathbb{R} \to \mathbb{R}$. Can the same definition work for a function $g : \mathbb{C} \to C$? What about a function $h : \mathbb{R}^n \to \mathbb{R}^n$,, $n = 2, 3, 4.....$ Does the usual definition of a derivative for such functions?

(iv) Let $f(z) = \sin z$. From the definition of the derivative, compute $f'(z)$.

(v) Let $f(x) = x \sin\left(\frac{1}{x^2}\right), x \neq 0$ and $f(0) = 0$. Show that $f'(0)$ exists, but $f''(x)$ does not exist. Thus a function of a real variable can be differentiable once, but not twice. What happens if we allow f to be a function of a complex variable?

(vi) What is the purpose of integration? Can the same definition of integral work for functions of a complex variable? Would the applications of integration be the same in the complex plane as on the real line?