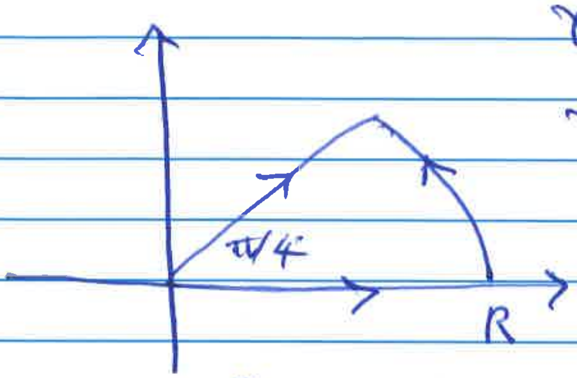
We use Cauchy's Theorem to calculate the Fresnel integrals ie $\int_{-\infty}^{\infty}\cos(t^2)\,dt = 2\int_0^{\infty}\cos(t^2)\,dt$

and $\int_{-\infty}^{\infty}\sin(t^2)\,dt = 2\int_0^{\infty}\sin(t^2)\,dt$

Recall that $\int_0^{\infty} f(x)\,dx = \lim_{R\to\infty}\int_0^R f(x)\,dx$.

We take $f(z) = e^{iz^2} = \cos(z^2) + i\sin(z^2)$

Let $\gamma = \gamma_1 + \gamma_2 - \gamma_3$



$\gamma_1(t) = t$, $t \in [0, R]$

$\gamma_2(t) = Re^{it}$, $t \in [0, \pi/4]$

$\gamma_3(t) = e^{\frac{i\pi}{4}}t$, $t \in [0, R]$

Then $\int_\gamma f(z)\,dz = 0$ by Cauchy's Theorem

Now $\int_{\gamma_1} f = \int_0^R e^{it^2}\,dt$

$$\int_{\gamma_2} f = \int_0^{\pi/4} iRe^{it}e^{iR^2e^{2it}}\,dt$$

$$\int_{\gamma_3} f = e^{\frac{i\pi}{4}}\int_0^R e^{-t^2}\,dt$$

$$\lim_{R\to\infty}\int_{\gamma_1} f = \int_0^{\infty}(\cos(t^2) + i\sin(t^2))\,dt$$

$$\lim_{R\to\infty}\int_{\gamma_3} f = e^{i\pi/4}\int_0^{\infty} e^{-t^2}\,dt = \frac{1}{2}\left(\frac{1}{\sqrt{2}} + \frac{i}{\sqrt{2}}\right)\sqrt{\pi}$$

Now $\left|\int_0^{\pi/4} iRe^{it}e^{iR^2e^{2it}}\,dt\right| \le \int_0^{\pi/4} Re^{-R^2\sin(2t)}\,dt$

Hence $\lim_{R\to\infty} \left| \int_{\gamma_2} f \right| \le \int_0^{\pi/4} \lim_{R\to\infty} e^{-R^2 \sin(2t)} R \, dt$

$= 0.$

Thus $\int_\gamma f = \int_{\gamma_1} f + \int_{\gamma_2} f - \int_{\gamma_3} f = 0$ implies

that as $R \to \infty$

$$\int_0^\infty \cos(t^2)\,dt + i \int_0^\infty \sin(t^2)\,dt$$

$$= \frac{\sqrt{\pi}}{2\sqrt{2}} + i \frac{\pi}{2\sqrt{2}}$$

And $\int_{-\infty}^{\infty} \cos(t^2)\,dt = \sqrt{\frac{\pi}{2}}$

$\int_{-\infty}^{\infty} \sin(t^2)\,dt = \sqrt{\frac{\pi}{2}}.$