Differential Equations Assignment 1. 2025 Autumn
Due April 10

(1) Solve the equation

$$x(1-2x^2y)y' + y = 3x^2y^2,$$

$y(1) = 1/2$, by setting $y = x^{-2}v$.

(2) Solve the Ricatti equation

$$y' + y^2 = x.$$

You will need Airy's equation.

(3) Use Variation of parameters to solve

$$x^2y'' + 4xy' - 10y = x^3 \sin x.$$

(4) Let p, q be analytic on the interval $I = (-a, a), a > 0$. Show that the IVP

$$y''(x) + p(x)y'(x) + q(x)y(x) = 0,$$

$y(0) = y'(0) = 0$ has $y = 0$ as a solution. Prove that this is the only solution on I.

(5) Solve the ODE

$$x^2y'' + x(2x-3)y' + 4y = 0,$$

by the method of Frobenius.