UNIVERSITY OF TECHNOLOGY, SYDNEY
SCHOOL OF MATHEMATICAL SCIENCES
35231 Differential Equations, Spring 2012
Class Test. Time Allowed: 55 minutes

(1) (a) Given that $y_1(x) = x$ is a solution of the DE

$$x^3 y'' - 3xy' + 3y = 0,$$

find a second linearly independent solution.

(b) Solve the equation $y'' + y = \tan x$ by variation of parameters. You may need $\sin^2 x = 1 - \cos^2 x$ and the integral $\int \sec x dx = \ln(\sec x + \tan x)$.

(2) Obtain the general solution of the equation

$$y'' - 2xy' + 6y = 0,$$

using a power series expansion of the form $y = \sum_{n=0}^{\infty} a_n x^n$.

(3) The Laplace transform of a suitable function $f(t)$ is given by $F(s) = \int_0^\infty e^{-st} f(t) dt$.

(a) Given that the Laplace transform of f is $F(s)$, obtain the Laplace transform of $tf(t)$ in terms of $F(s)$.

(b) Calculate the Laplace transform of $t\sin(2t)$.

(c) Suppose that y, y' are integrable functions and $y(0) = 2$. express the Laplace transform of y' in terms of the Laplace transform of y.

You may need the following information.

If $\mathcal{L}$ denotes the Laplace transform, then

$$\mathcal{L}(t^n) = \frac{n!}{s^{n+1}}.$$

$$\mathcal{L}(e^{at}) = \frac{1}{s-a}.$$

$$\mathcal{L}(\sin at) = \frac{a}{s^2+a^2}.$$

$$\mathcal{L}(\cos at) = \frac{s}{s^2+a^2}.$$

If $y'' + p(x)y' + q(x)y = R(x)$ then a particular solution takes the form $y = u(x)y_1(x) + v(x)y_2(x)$, where y_1 and y_2 are solutions of the homogeneous problem and

$$\begin{aligned} u'y_1 + v'y_2 &= 0 \\ u'y_1' + v'y_2' &= R(x). \end{aligned}$$

Bessel's differential equation $t^2u'' + tu' + (t^2 - \alpha^2)u = 0$ may be transformed into the equation

$$x^2y'' + (1 - 2s)xy' + ((s^2 - r^2\alpha^2) + a^2r^2x^{2r})y = 0$$

under the change of variables $t = ax^r$ and $y(x) = x^s u(t)$.