$$a_8 = \frac{2(6-3)}{8\times 7} a_6 = \frac{2^4(-1)(-3)(1)(3)}{8\times 7\times 6\times 5\times 4\times 3\times 2} a_0 = \frac{2^4\times 3^2}{8!} a_0$$ 3 for first few even coefficients

$$a_{10} = \frac{2(8-3)}{10\times 9} a_8 = \frac{2^5 3^2\times 5}{10!} a_0$$

$$a_{12} = \frac{2^6 3^2\times 5\times 7}{12!} a_0 \quad \text{etc.}$$

$\left[ \begin{array}{l} ak+b=3, k=4 \\ ak+b=5, k=5 \\ \therefore a=2, b=-5 \end{array} \right]$

So $a_{2k} = \dfrac{2^k 3^2\times 5\times \ldots (2k-5)}{(2k)!} a_0, \ k \geq 3,$ 2

Thus for odds

$$y = a_1 x - \frac{2}{3} a_1 x^3 = a_1\left(x - \frac{2}{3}x^3\right)$$ 2 mark

even

$$y = a_0\left(1 - 3x^2 + \frac{1}{2}x^4 + \frac{1}{30}x^6 + \ldots\right) (*)$$ 2

$$= a_0\left(1 - 3x^2 + \frac{1}{2}x^4 + \frac{1}{30}x^6 + \sum_{k=4}^{\infty} \frac{2^k 3^2\times 5\times \ldots (2k-5)}{(2k)!} x^{2k}\right)$$ 2

or equivalent (could start series at $k=3$, and so $\frac{1}{30}x^6$ would be in sum)

Give 4 marks if only this, but not (*)

Give 3 marks if only (*) is given

3(a) $$\int_0^\infty t f(t) e^{-st}\,dt = -\int_0^\infty f(t) \frac{d}{ds}(e^{-st})\,dt$$ ~~3~~4

$$= -\frac{d}{ds}\int_0^\infty f(t) e^{-st}\,dt$$

$$= -\frac{d}{ds} F(s)$$

(b) $$\int_0^\infty t\sin(2t) e^{-st}\,dt = -\frac{d}{ds}\int_0^\infty \sin(2t) e^{-st}\,dt$$ 3

$$= -\frac{d}{ds}\left(\frac{2}{s^2+4}\right) = \frac{4s}{(s^2+4)^2}$$