UNIVERSITY OF TECHNOLOGY SYDNEY
SCHOOL OF MATHEMATICAL AND PHYSICAL SCIENCES
37335 Differential Equations, Spring 2016
Class Test. Time Allowed: 55 minutes

(1) Given that $y_1(x) = x$ is a solution of the DE

$$x^2y'' - 4xy' + 4y = 0,$$

find the general solution of the equation

$$x^2y'' - 4xy' + 4y = x^3.$$

(2) Obtain the general solution of the equation

$$y'' + (1 - x)y' + y = 0,$$

using a power series expansion of the form $y = \sum_{n=0}^{\infty} a_n x^n$.

(3) The Laplace transform of a suitable function $f(t)$ is given by $F(s) = \mathcal{L}(f)(s) = \int_0^\infty e^{-st} f(t)dt$.

(a) Invert the Laplace transform

$$F(s) = \frac{s+2}{(s+4)(s^2+16)}.$$

You will need the table at the back and to find A, B, C such that

$$\frac{s+2}{(s+4)(s^2+16)} = \frac{A}{s+4} + \frac{Bs+C}{s^2+16}.$$

(b) Given that $F(s) = \mathcal{L}(f)(s)$, obtain the Laplace transform of $h(t) = tf(t)e^{-t}$ in terms of F. Hint: What is the derivative $\frac{d}{ds}e^{-(s+1)t}$?

Table of Laplace transforms

$$\mathcal{L}(t^n) = \frac{n!}{s^{n+1}}$$
$$\mathcal{L}(e^{-at}) = \frac{1}{s+a}$$
$$\mathcal{L}(\sin(at)) = \frac{a}{s^2+a^2}$$
$$\mathcal{L}(\cos(at)) = \frac{s}{s^2+a^2}$$
$$\mathcal{L}(J_0(t)) = \frac{1}{\sqrt{1+s^2}}.$$

Variation of parameters

Given that y_1 and y_2 are solutions of the ODE

$$y''(x) + b(x)y'(x) + c(x)y(x) = 0,$$

we seek a particular solution of the ODE

$$y''(x) + b(x)y'(x) + c(x)y(x) = f(x),$$

by looking for solutions of the form $y_p(x) = u(x)y_1(x) + v(x)y_2(x)$. The functions u and v must satisfy

$$u'y_1 + v'y_2 = 0,$$
$$u'y_1' + v'y_2' = f(x).$$