UNIVERSITY OF TECHNOLOGY SYDNEY
SCHOOL OF MATHEMATICAL AND PHYSICAL SCIENCES
37335 Differential Equations, Spring 2019
Class Test A. Time Allowed: 50 minutes

(1) Given that $y_1(x) = x$ is a solution of the DE

$$x^2y'' + 4xy' - 4y = 0,$$

find the general solution of the equation

$$x^2y'' + 4xy' - 4y = 3x^3.$$

(2) Obtain the general solution of the equation

$$y'' - 2xy' + 4y = 0,$$

using a power series expansion of the form $y = \sum_{n=0}^{\infty} a_n x^n$.

(3) (i) The Laplace transform of f is defined to be

$$F(s) = \mathcal{L}(f) = \int_0^{\infty} e^{-st} f(t)dt.$$

Prove that

$$\mathcal{L}(f') = sF(s) - f(0).$$

(ii) Use the Laplace transform to solve the ODE

$$y' + 4y = \sin x,\ \ y(0) = 2.$$

Table of Laplace transforms

$$\mathcal{L}(t^n) = \frac{n!}{s^{n+1}}$$
$$\mathcal{L}(e^{-at}) = \frac{1}{s+a}$$
$$\mathcal{L}(\sin(at)) = \frac{a}{s^2+a^2}$$
$$\mathcal{L}(\cos(at)) = \frac{s}{s^2+a^2}$$
$$\mathcal{L}(J_0(t)) = \frac{1}{\sqrt{1+s^2}}.$$

Variation of parameters

Given that y_1 and y_2 are solutions of the ODE

$$y''(x) + b(x)y'(x) + c(x)y(x) = 0,$$

we seek a particular solution of the ODE

$$y''(x) + b(x)y'(x) + c(x)y(x) = f(x),$$

by looking for solutions of the form $y_p(x) = u(x)y_1(x) + v(x)y_2(x)$. The functions u and v must satisfy

$$u'y_1 + v'y_2 = 0,$$
$$u'y_1' + v'y_2' = f(x).$$

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Class Test B. Time Allowed: 50 minutes

(1) Given that $y_1(x) = x$ is a solution of the DE

$$x^2y'' + 3xy' - 3y = 0,$$

find the general solution of the equation

$$x^2y'' + 3xy' - 3y = 5x^4.$$

(2) Obtain the general solution of the equation

$$y'' - 4xy' + y = 0,$$

using a power series expansion of the form $y = \sum_{n=0}^{\infty} a_n x^n$.

(i) The Laplace transform of f is defined to be

$$F(s) = \mathcal{L}(f) = \int_0^{\infty} e^{-st} f(t)dt.$$

Prove that

$$\mathcal{L}(f') = sF(s) - f(0).$$

(ii) Use the Laplace transform to solve the ODE

$$y' - 2y = \cos x, \; y(0) = 1.$$

Table of Laplace transforms

$$\begin{aligned}
\mathcal{L}(t^n) &= \frac{n!}{s^{n+1}} \\
\mathcal{L}(e^{-at}) &= \frac{1}{s+a} \\
\mathcal{L}(\sin(at)) &= \frac{a}{s^2+a^2} \\
\mathcal{L}(\cos(at)) &= \frac{s}{s^2+a^2} \\
\mathcal{L}(J_0(t)) &= \frac{1}{\sqrt{1+s^2}}.
\end{aligned}$$

Variation of parameters

Given that y_1 and y_2 are solutions of the ODE

$$y''(x) + b(x)y'(x) + c(x)y(x) = 0,$$

we seek a particular solution of the ODE

$$y''(x) + b(x)y'(x) + c(x)y(x) = f(x),$$

by looking for solutions of the form $y_p(x) = u(x)y_1(x) + v(x)y_2(x)$. The functions u and v must satisfy

$$\begin{aligned}
u'y_1 + v'y_2 &= 0, \\
u'y_1' + v'y_2' &= f(x).
\end{aligned}$$

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Class Test C. Time Allowed: 50 minutes

(1) Given that $y_1(x) = x$ is a solution of the DE
$$x^2y'' + 7xy' - 7y = 0,$$
find the general solution of the equation
$$x^2y'' + 7xy' - 7y = x^6.$$

(2) Obtain the general solution of the equation
$$y'' - xy' + 4y = 0,$$
using a power series expansion of the form $y = \sum_{n=0}^{\infty} a_n x^n$.

(i) The Laplace transform of f is defined to be
$$F(s) = \mathcal{L}(f) = \int_0^\infty e^{-st} f(t) dt.$$
Prove that
$$\mathcal{L}(f') = sF(s) - f(0).$$

(ii) Use the Laplace transform to solve the ODE
$$y' + 5y = \sin(2x), \; y(0) = 1.$$

Table of Laplace transforms

$$\mathcal{L}(t^n) = \frac{n!}{s^{n+1}}$$
$$\mathcal{L}(e^{-at}) = \frac{1}{s+a}$$
$$\mathcal{L}(\sin(at)) = \frac{a}{s^2+a^2}$$
$$\mathcal{L}(\cos(at)) = \frac{s}{s^2+a^2}$$
$$\mathcal{L}(J_0(t)) = \frac{1}{\sqrt{1+s^2}}.$$

Variation of parameters

Given that y_1 and y_2 are solutions of the ODE

$$y''(x) + b(x)y'(x) + c(x)y(x) = 0,$$

we seek a particular solution of the ODE

$$y''(x) + b(x)y'(x) + c(x)y(x) = f(x),$$

by looking for solutions of the form $y_p(x) = u(x)y_1(x) + v(x)y_2(x)$. The functions u and v must satisfy

$$u'y_1 + v'y_2 = 0,$$
$$u'y_1' + v'y_2' = f(x).$$