1. Elementary Integrals and Differential Equations

(1) Evaluate the following integrals

(i)
$$\int x \sin x dx.$$

(ii)
$$\int \tan x dx.$$

(iii)
$$\int \sec x dx.$$

(This is hard).

(iv)
$$\int \frac{x dx}{(x+1)(x+2)}.$$

(2) Solve the following differential equations. Find the exact solution if initial values are given.

(i)
$$\frac{dy}{dx} + 4y = 0,\ y(0) = 2$$

(ii)
$$y' + 4y = 6x,\ y(0) = 5.$$

(iii)
$$\frac{d^2y}{dt^2} + 3\frac{dy}{dt} + 2y = 0,\ y(0) = 1, y'(0) = 2.$$

(iv)
$$\frac{d^2y}{dx^2} + 4y = \sin x.$$

(v)
$$y'' + 5y' + 25y = 0,\ y(0) = 1, y'(0) = 2.$$