UTS

Stochastic Processes and Financial Mathematics

Lab/Tutorial 10

This lab is not assessed.

Question 1

Let the geometric Brownian Motion (GBM)

$$S_t = S_0 \exp\big((r - q - \sigma^2/2)t + \sigma B_t\big), \qquad 0 \le t \le T,$$

where the continuous risk-free rate $r = 0.01$, the continuous dividend yield is $q = 0.02$, volatility $\sigma = 0.5$, maturity $T = 1$ and B_t is a standard Brownian motion.

In this lab we will price various options using the PDE

$$\frac{\partial}{\partial t}V(t,x) + (r-q)x\frac{\partial}{\partial x}V(t,x) + \frac{1}{2}\sigma^2 x^2 \frac{\partial^2}{\partial x^2}V(t,x) = rV(t,x).$$

First consider the European vanilla call option with price

$$V(t,x) = e^{-r(T-t)}E[\max(S_T - 20,0)|S_t = x].$$

(a) Implement an explicit finite difference scheme to approximate the price $V(t,S_t)$ using step sizes $\Delta t = 1/250$ and $\Delta x = 1/4$ on a mesh defined by $0 \le t \le 1, 0 \le x \le 60$. What do you notice?

i. Provide (well, attempt to!) a graph of the pricing surface $V(t,S_t)$. What do you notice?

(b) Implement the "Crank-Nicholson" finite difference scheme to approximate the price $V(t,S_t)$ using step sizes $\Delta t = 1/250$ and $\Delta x = 1/4$ on a mesh defined by $0 \le t \le 1, 0 \le x \le 60$.

i. Quote the price $V(0,26)$.

ii. Provide a graph of the pricing surface $V(t,S_t)$.

iii. Compare the finite difference price to the price given by the Black Scholes equation.

Now consider the American vanilla call option with price

$$V(t,x) = \max_{t\le\tau\le T} e^{-r(\tau-t)} E[\max(S_\tau - 20{,}0)|S_t = x].$$

An American option can be exercised at any time, unlike a European option that can only be exercised at maturity.

(c) Implement a "Crank-Nicholson" finite difference scheme to approximate the price $V(t,S_t)$ of the American vanilla call using step sizes $\Delta t = 1/250$ and $\Delta x = 1/4$ on a mesh defined by $0 \le t \le 1, 0 \le s \le 60$.

At each time step you will a candidate price $V^*(t,S_t)$ on the finite difference mesh. This candidate price needs to be compared to the early exercise condition $\max(S_t - 20{,}0)$ with the maximum of these selected for that point on the mesh.

That is, at each point on the finite difference mesh set

$$V(t,S_t) = \max\big(\max(S_t - 20{,}0), V^*(t,S_t)\big).$$

i. Quote the price $V(0{,}26)$.

ii. Provide a graph of the pricing surface $V(t,S_t)$.

Finally, consider the European up and out barrier call with price

$$V(t,x) = e^{-r(T-t)} E\left[\max(S_T - 20{,}0) I\left(\max_{0\le t\le T} S_t < 30\right)|S_t = x\right].$$

(d) Implement a "Crank-Nicholson" finite difference scheme to approximate the price $V(t,S_t)$ of the European up and out barrier call using step sizes $\Delta t = 1/250$ and $\Delta x = 1/4$ on a mesh defined by $0 \le t \le 1, 0 \le s \le 30$.

At each time step you will need to set the boundary condition $V(t,30) = 0$ on your finite difference mesh.

i. Quote the price $V(0{,}26)$.

ii. Provide a graph of the pricing surface $V(t,S_t)$.

(e) Optional extra work. Implement the finite difference scheme in R.