UTS

Stochastic Processes and Financial Mathematics (37363)

Lab/Tutorial 6

This lab/tutorial is assessed and marked from 24.

To obtain maximum possible marks show all steps necessary to derive answers and explain your reasoning where necessary. If only final line of answer is provided then only 1/3 marks will be awarded for the question part.

Unless otherwise stated, you may use computational software for calculations, but code and output does not constitute an answer. If only this is provided then 1/3 marks will be awarded for the question part.

Where marked as such, you must use R to answer computation questions.

Include images of all R code/output you refer to in your answers.

If relevant R code/output is not provided, then only 1 out of 3 possible marks will be awarded for the question part irrespective of the answer given.

Please write up your answers to these questions and upload your work in PDF format to Canvas.

Due by 23:59 Tuesday 2nd April 2025.

Question 1. MC simulation and option pricing

Consider the geometric Brownian Motion (GBM)

$$S_t = S_0 \exp\left(\left(r - \frac{\sigma^2}{2}\right)t + \sigma B_t\right), \qquad 0 \le t \le T,$$

where B_t a standard Brownian motion and $S_0 = 50, r = 3/100, \sigma = 4/10$ and $T = 5$.

(a) Find $E[S_t]$ and $\mathrm{var}(S_t)$ **[3 marks]**.

Hint 1. Note that

$$\begin{aligned} E[S_t] &= E\left[S_0 \exp\left(\left(r - \frac{\sigma^2}{2}\right)t + \sigma B_t\right)\right] \\ &= S_0 \exp\left(\left(r - \frac{\sigma^2}{2}\right)t\right) E[\exp(\sigma B_t)] \end{aligned}$$

and then use Gaussian MGF.

(b) Show that.

$$\begin{aligned} S_{t+\Delta t} &= S_t \exp\left(\left(r - \frac{\sigma^2}{2}\right)\Delta t + \sigma(B_{t+\Delta t} - B_t)\right) \\ &\stackrel{d}{=} S_t \exp\left(\left(r - \frac{\sigma^2}{2}\right)\Delta t + \sigma B_{\Delta t}\right) \end{aligned}$$

where $\stackrel{d}{=}$ indicates equality in distribution **[not assessed]**.

(c) Taking 1000 equally-spaced time steps, use R to simulate a path of S_t for $0 \le t \le T$ **[3 marks]**.

Hint 2. Use the recursion formula derived in (b)

$$S_{t+\Delta t} = S_t \exp\left(\left(r - \frac{\sigma^2}{2}\right)\Delta t + \sigma B_{\Delta t}\right)$$

where $\Delta t = T/1000$ is the increment of time.

Hint 3. Use the R functions `rnorm`, `for` and `plot`.

(d) The price of a European vanilla call option at time $t = 0$ is given by

$$C = e^{-rT} E[\max(S_T - K, 0)].$$

Taking $n = 10^5$ and $K = 45$, use R to calculate the crude Monte Carlo estimate

$$\begin{aligned} C_n &= \frac{e^{-rT}}{n} \sum_{k=1}^{n} \max\left(S_T^{(k)} - K, 0\right) \\ &= \frac{e^{-rT}}{n} \sum_{k=1}^{n} \max\left(S_0 \exp\left(\left(r - \frac{\sigma^2}{2}\right) T + \sigma B_T^{(k)}\right) - K, 0\right) \end{aligned}$$

where $S_T^{(k)}, B_T^{(k)}$ are random samples of S_T and B_T respectively. Also calculate the standard error of the estimated price **[3 marks]**.

Question 2. Constructing correlated BM

Let $(B_t, W_t)^T, t \geq 0$, be 2D standard BM with independent components.

(a) Verify that

$$Y_t = \rho B_t + \sqrt{1 - \rho^2}\, W_t, \qquad |\rho| < 1, \qquad t \geq 0,$$

is another standard BM **[3 marks]**.

(b) Show that $\text{corr}(Y_t, B_t) = \rho$ **[3 marks]**.

Question 3. Checking for stationarity

Consider the process

$$Y_t = 2X_{t+1} - 3X_{t-1}, \qquad t \in \mathbb{Z},$$

where the $X_t \sim N(0,1)$ and independent from each other. Determine if the following processes are weakly stationary.

(a) $Z_t = Y_1 Y_{t+3},\ t \in \mathbb{Z}$ **[3 marks]**.

Hint 1. Consider cases $t = -4, -2, 0$ and otherwise.

(b) $Z_t = \cos(t) Y_1 + \sin(t) Y_3,\ t \in \mathbb{Z}$ **[not assessed]**.

Question 4. S&P 200 returns

Refer to R code files “37363_LabTutorial6_Autumn2025_class.R”.

(a) Using R, Plot the closing prices **[3 marks]**.

(b) Generate the log returns using code below.

```
log_returns <- diff(log(close))
```

Using R, plot the empirical distribution function of the log returns (use R functions `ecdf` and `plot`) and superimpose the normal distribution function fitted to the log returns (use R functions `curve` and `pnorm`). Do the log returns appear normally distributed **[3 marks]**?

Hint 1. Calculate sample mean and sample standard deviation of the log returns and pass to R function `pnorm`.