UTS

Stochastic Processes and Financial Mathematics

Lab/Tutorial 9

This lab/tutorial is assessed and marked from 24.

To obtain maximum possible marks show all steps necessary to derive answers and explain your reasoning where necessary. If only final line of answer is provided then only 1/3 marks will be awarded for the question part.

Unless otherwise stated, you may use R (or Mathematica) for calculations, but code and output does not constitute an answer. If only this is provided then 1/3 marks will be awarded for the question part.

Include images of all code/output you refer to in your answers. If relevant code/output is not provided, then only 1 out of 3 possible marks will be awarded for the question part irrespective of the answer given.

Please write up your answers to these questions and upload your work in PDF format to Canvas.

Due by 23:59 Tuesday 6th May 2025.

Question 1 [9 marks]. Compound Poisson process

Consider a stock market that operates for 6 hours per day, 5 days per week. On this market there are two stocks, A and B. Stock A trades according to a Poisson process with an average of one transaction per hour. Stock B trades according to a Poisson process with an average of two transaction per hour. Suppose that transactions of both stocks occur independently of each other.

Define appropriate stochastic processes in answering these questions.

(a) Suppose 20 transactions of stock A occur in the first week of the year and 30 transactions of stock A occur in the second week. Calculate the probability of stock A trading 110 times in the first four weeks of the year (express final answer to at least 6 decimal places) **[3 marks]**.

(b) Suppose 250 transactions of stock B occur in the first five weeks of the year. Calculate the probability of stock A trading 30 times and stock B trading 50 times in the first week of the year (express final answer to at least 6 decimal places) **[3 marks]**.

Assume that for each transaction of stock A, the volume traded is one (with probability 0.1), two (with probability 0.7) or three (with probability 0.2).

Assume further that for each transaction of stock B, the volume traded is one (with probability 0.2), three (with probability 0.3) or five (with probability 0.5).

(c) Calculate the variance of the total volume traded at the exchange over the first two weeks of the year **[3 marks]**.

Question 2 [15 marks]. Merton jump-diffusion model

A common model for a financial asset is the Merton jump-diffusion process

$$S_t = S_0 e^{X_t}, t \geq 0,$$

where

$$X_t = \mu t + \sigma B_t + \sum_{k=1}^{N_t} Y_k,$$

$$\mu = r - \frac{1}{2}\sigma^2 + \lambda\left(1 - e^{m+\theta^2/2}\right),$$

with B_t a standard Wiener process, N_t a Poisson process with intensity λ, $Y_k \sim N(m, \theta^2)$ and independent and the constants $\mu, m \in \mathbb{R}$ and $\sigma, \theta > 0$.

(a) Find $E[S_t]$, making sure to simplify your answers as far as possible **[3 marks]**.

Hint 1. Note that B_t, N_t and the Y_k are independent.

(b) Find $\text{var}(S_t)$ **[3 marks]**.

(c) Find $\text{cov}(X_t, X_s)$ for $0 < s < t$ **[3 marks]**.

(d) Show that

$$S_{t+\Delta t} \overset{d}{=} S_t \exp\left(\mu \Delta t + \sigma B_{\Delta t} + \sum_{k=N_t+1}^{N_{t+\Delta t}} Y_k\right).$$

[3 marks]

(e) Using R (not Mathematica) and taking 1000 equally-spaced time steps, plot a path of S_t for $0 \leq t \leq 1$ using parameters $S_0 = 10, r = 0.04, \sigma = 0.5, \lambda = 15, m = -0.2$ and $\theta = 0.5$ **[3 marks]**.

Hint 2. Use result from (d).

Hint 3. For each time increment Δt, generate the number of jumps as a $N_{t+\Delta t} - N_t$ as a Poisson$(\lambda \Delta t)$ RV.

Hint 4. For each time increment Δt, generate the jump sizes Y_k as $N(m, \theta^2)$ RVs. Do this for the number of jumps found in Hint 3.

Hint 5. Add the sum of the jump sizes found in Hint 4 to $\mu \Delta t + \sigma(B_{t+\Delta t} - B_t)$ (see previous labs for simulating Brownian motion).