Class Test 1

St.Id: Name:

There are three questions in this test, total time 45 min.

Q1. Consider the following LP problem: (6 marks)

$$\max z = 2x_1 + x_2$$

s.t. $-x_1 + x_2 \le 10$ → $x_2 \le 10 + x_1$

$0.6x_1 + x_2 \le 15$ → $x_2 \le 15 - 0.6x_1$

$-2.2x_1 + x_2 \ge 1$ → $x_2 \ge 1 + 2.2x_1$

$x_1, \ x_2 \ge 0$

a) Graph the feasible region of this LP

b) Find its extreme points – please specify their coordinates

c) What is the optimal value of the objective function and what is the optimal solution?

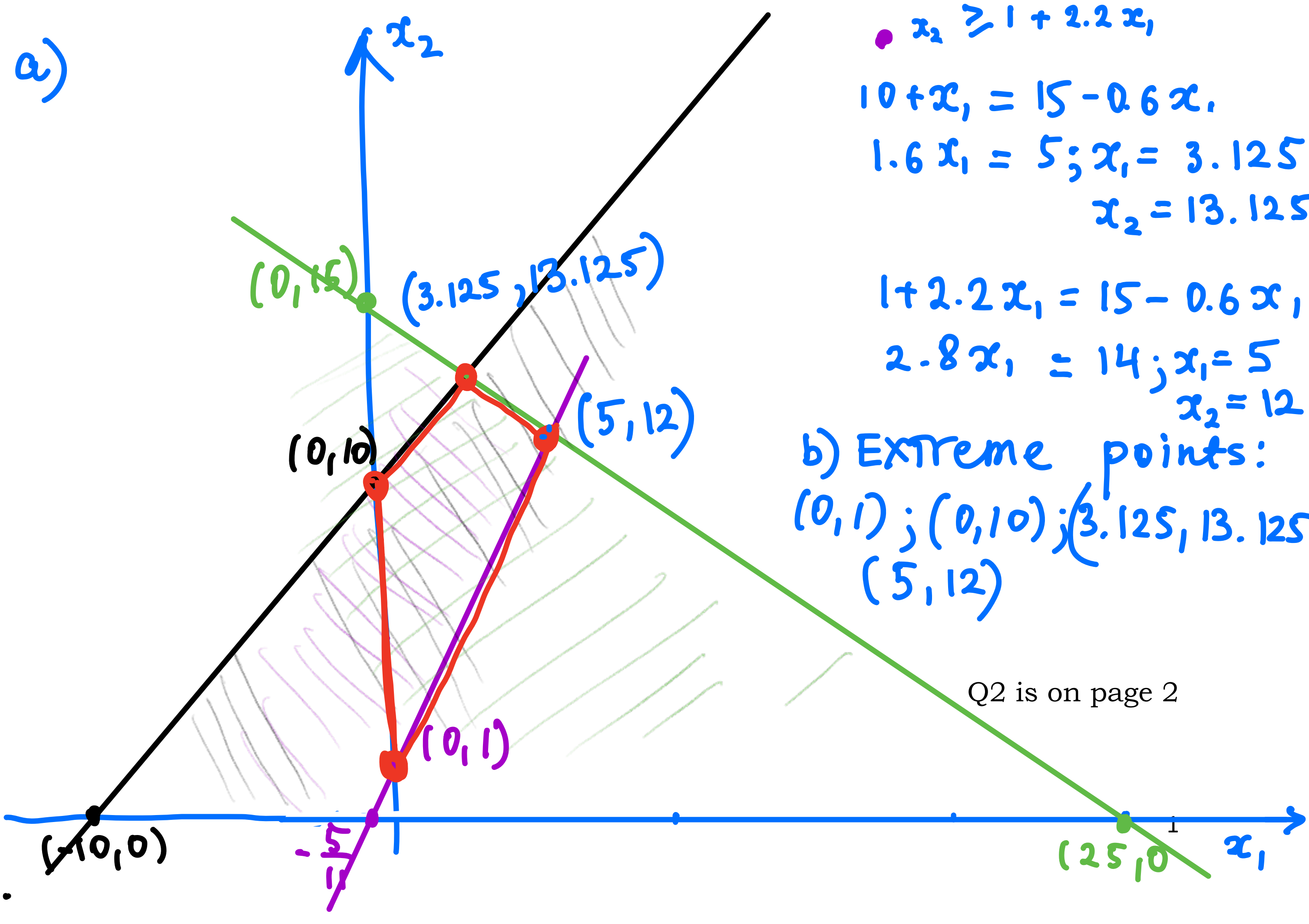
Please provide your solution below:

$x_2 \le 10 + x_1$

$x_2 \le 15 - 0.6x_1$

$x_2 \ge 1 + 2.2x_1$

a)



$10 + x_1 = 15 - 0.6x_1$

$1.6x_1 = 5; x_1 = 3.125$

$x_2 = 13.125$

$1 + 2.2x_1 = 15 - 0.6x_1$

$2.8x_1 = 14; x_1 = 5$

$x_2 = 12$

b) Extreme points:

(0,1); (0,10); (3.125, 13.125) (5,12)

Q2 is on page 2

c) $z = 2x_1 + x_2$:

$z(0,1) = 1$ $\quad$ $z(3.125, 13.125) = 19.375$

$z(0,10) = 10$ $\quad$ $z(5,12) = 22$

$\max z = 22$;

$$x = \begin{pmatrix} 5 \\ 12 \end{pmatrix}$$

Q2. Consider the following LP problem: (6 marks)

$$\min\ z = 2x_1 - 5x_2$$
$$\text{s.t.} \quad 3x_1 + 8x_2 \leq 12$$
$$2x_1 + 3x_2 \leq 6$$
$$x_1,\ x_2 \geq 0$$

a) Write the problem in a standard form
b) Specify the initial basis
c) Solve the problem by the Simplex method in tabular form, show the detailed working .
d) Specify the optimal value of objective function and the optimal solution.

Please provide your solution below:

a) Standard form:

$\min z = 2x_1 - 5x_2 \rightarrow z - 2x_1 + 5x_2 = 0.$

s.t. $3x_1 + 8x_2 + s_1 = 12$

$2x_1 + 3x_2 + s_2 = 6$

$x_1\ x_2\ s_1\ s_2 \geq 0.$

b) $x_{B_0} = (s_1\ s_2)$

	x_1	x_2	s_1	s_2	RHS
z	-2	5	0	0	0
s_1	3	8	1	0	12
s_2	2	3	0	1	6
z	$-31/8$	0	$-5/8$	0	$-15/2$
x_2	$3/8$	1	$1/8$	0	$3/2$
s_2	$\frac{7}{8}$	0	$-3/8$	1	$\frac{3}{2}$

Ratio:
$12/8 < 2$
$6/3 = 2$

$R_0' = R_0 - 5R_1'$
$R_1' = \frac{R_1}{8}$
$R_2' = R_2 - 3R_3'$

Optimal Tableau, as all $\hat{c}_N < 0$

$z^* = \frac{-15}{2}$

$x_1 = 0$

$x_2 = 3/2$

Q3 is on page 3

Q3. Consider the set of vectors $x = \binom{x_1}{x_2}$ satisfying (3 marks)

$$2x_1 - 5x_2 \leq 15, \qquad x_1, x_2 \geq 0$$

a) Write the inequality above in the form $ax \leq b$, where a is a vector and b is a constant

b) Prove that this set is convex.

Please provide your solution below:

a) $(2, 5)\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \leq 15$

b) Let $x' = \begin{pmatrix} x_1' \\ x_2' \end{pmatrix}$ and $x'' = \begin{pmatrix} x_1'' \\ x_2'' \end{pmatrix}$:

$(2,5)x' \leq 15 \qquad (2,5)x'' \leq 15$

and $\tilde{x} = \alpha x' + (1-\alpha)x''$, $0 < \alpha < 1$

Then $(2,5)\tilde{x} = \alpha(2,5)x' + (1-\alpha)(2,5)x'' \leq$

$\leq \alpha * 15 + (1-\alpha) * 15 = 15$

Hence $(2,5)\tilde{x} \leq 15 \rightarrow$ the set is convex

Table of formulae for Class Test 1

- *Convex set:* A set S in n-dimensional space is *convex* if for any two points $x^{(1)}$ and $x^{(2)}$ from S, any point of the line segment connecting $x^{(1)}$ and $x^{(2)}$ also belongs to S. In other words, a set S is a convex set if the line segment joining any pair of points in S is wholly contained in S: for any two points $x^{(1)}$ and $x^{(2)}$ in S and $\alpha \in (0,1)$, $x^* = \alpha x^{(1)} + (1-\alpha)\, x^{(2)}$: $x^* \in S$

Full Simplex tableau:

basis	x_N	x_B	rhs
z	$c_B^T B^{-1} N - c_N^T$	0^T	$c_B^T B^{-1} b$
x_B	$B^{-1} N$	I	$B^{-1} b$