Class Test 2

St.Id: Name:

There are three questions in this test, total time 50 min.

Q1. Consider the following LP problem: 10 marks

$$\min z = -x_1 - 2x_2$$
$$s.t. \quad 3x_1 + 2x_2 \le 24$$
$$x_1 + x_2 \ge 4$$
$$x_1, \; x_2 \ge 0$$

Use one of Simplex methods in tabular form to solve the problem:

a) Specify the initial basis
b) Write the initial tableau in a canonical form
c) Solve the problem with the chosen Simplex method
d) State the optimal value of objective function and the optimal solution.

1. Using Dual Simplex method:

$\min z = -x_1 - 2x_2$

s.t. $3x_1 + 2x_2 + s_1 = 24$

$x_1 + x_2 - e_2 = 4 \longrightarrow$ not canonical $\quad -x_1 - x_2 + e_2 = -4$ 1

$x_1, x_2, s_1, e_2 \ge 0$

$x_{B_0} = (s_1, e_2)$ 1

	x_1	x_2	s_1	e_2	RHS
z	1	2	0	0	0
s_1	3	2	1	0	24
e_2	-1	-1	0	1	-4

← Initial Tableau in canonical form

1

Use the next page to provide the solution:

Please provide your solution for Q1 here:

1

	x_1	x_2	s_1	e_2	RHS
z	1	2	0	0	0
s_1	3	2	1	0	24
e_2	$\boxed{-1}$	−1	0	1	−4 < 0
z	0	1	0	1	−4
s_1	0	−1	1	3	12
x_1	1	$\boxed{1}$	0	−1	4
z	0	0	0	2	−8
s_1	0	0	1	$\boxed{2}$	16
x_2	1	1	0	−1	4
z	0	0	−1	0	−24
e_2	0	0	1/2	1	8
x_2	1	1	1/2	0	12

Ratio: $\left|\frac{1}{-1}\right| < \left|\frac{2}{-1}\right|$

1 leaving

$R_0' = R_0 + R_2$
$R_1' = R_3 - 3R_2'$
$R_2' = R_2 \times (-1)$

1

$R_0 = R_0 - R_2$
$R_1' = R_1 + R_2$
$R_2' = R_2$

1

$R_0' = R_0 - R_1$
$R_1' = R_1/2$
$R_2' = R_2 + R_1'$

1

Optimal, as all $\hat{c}_N \leq 0$ 1

$z^* = -24$
$x_1^* = 0$
$x_2 = 12$

1

Q2 is on page 3

2. Using Big M

$$\min z = -x_1 - 2x_2$$
$$\text{s.t.} \quad 3x_1 + 2x_2 \leq 24 \rightarrow$$
$$x_1 + x_2 \geq 4$$
$$x_1,\ x_2 \geq 0$$

$\min z = -x_1 - 2x_2 + Ma_2$ 1

s.t. $3x_1 + 2x_2 + s_1 = 24$

R2: $x_1 + x_2 - e_2 + a_2 = 4$

$x_1, x_2, e_2, a_2 \geq 0.$ 1

$x_{B_0} = (s_1, a_2)$ 1

$z + x_1 + 2x_2 - Ma_2 = 0 \rightarrow$ not canonical

$+ MR_2$:

$$\begin{array}{l} Mx_1 + Mx_2 - Me_2 + Ma_2 = 4M \\ \hline z + (M+1)x_1 + (M+2)x_2 - Me_2 = 4M \end{array}$$

1 making canonical

Initial tableau in canonical form 1 in canonical

	x_1	x_2	s_1	e_2	a_2	RHS
z	$M+1$	$M+2$	0	$-M$	0	$4M$
s_1	3	2	1	0	0	24
a_2	1	$\boxed{1}$	0	-1	1	4

$24/2 > 4/1$ 1

	x_1	x_2	s_1	e_2	a_2	RHS
z	-1	0	0	2	$-(M+2)$	-8
s_1	1	0	1	$\boxed{2}$	-2	16
x_2	1	1	0	-1	1	4

$R_0' = R_0 - (M+2)R_2$

$R_1' = R_1 - 2R_2$ 1

$R_2' = R_2$

	x_1	x_2	s_1	e_2	a_2	RHS
z	-2	0	-1	0	$-M$	-24
e_2	$1/2$	0	$1/2$	1	-1	8
x_2	$3/2$	1	$1/2$	0	0	12

$R_0' = R_0 - R_1$ 1

$R_1' = R_1 / 2$

$R_2' = R_2 + R_1'$

Optimal Tableau, as all $\hat{c}_N < 0$ 1

$z^* = -24$

$x_1^* = 0$ 1

$x_2^* = 12$

3. Using Two-phase Simplex

$$\min z = -x_1 - 2x_2$$
s.t. $3x_1 + 2x_2 \leq 24$
$x_1 + x_2 \geq 4$
$x_1,\ x_2 \geq 0$

$\rightarrow$

$\min z = -x_1 - 2x_2$
s.t. $3x_1 + 2x_2 + S_1 = 24$
R2: $x_1 + x_2 - e_2 + a_2 = 4$
$x_1\ x_2\ e_2\ a_2 \geq 0.$
$(*)$

1

Phase 1

$\min w = a_2$
s.t. $(*)$

✓ 1

$x_{B_0} = (S_1\ a_2)$ $\quad w - a_2 = 0 \rightarrow$ not canonical

$$\begin{array}{r} w - a_2 = 0 \\ + R_2:\ x_1 + x_2 - e_2 + a_2 = 4 \\ \hline w + x_1 + x_2 - e_2 = 4 \end{array}$$

0.5

	x_1	x_2	s_1	e_2	a_2	RHS
w	1	1	0	-1	0	4
s_1	3	2	1	0	0	24
a_2	$\boxed{1}$	1	0	-1	1	4
w	0	0	0	0	-1	0
s_1	0	-1	1	3	-3	12
x_1	1	1	0	-1	1	4

$24/3 > 4/1$

$R_0' = R_0 - R_2$
$R_1' = R_1 - 3R_2$
$R_2' = R_2$

1

optimal, $w^* = 0 \rightarrow$ proceed to phase 2: 1

Phase 2

$z + x_1 + 2x_2 = 0$

	x_1	x_2	s_1	e_2	a_2	RHS
z	1	2	0	0		0
s_1	0	-1	1	3	-3	12
x_1	1	1	0	-1	1	4

NOT canonical 0.5
as x_1 is in basis
$R_0' = R_0 - R_2$

	x_1	x_2	s_1	e_2	RHS
z	0	1	0	1	-4
s_1	0	-1	1	3	12
x_1	1	$\boxed{1}$	0	-1	4

$R_0' = R_0 - R_2$
$R_1' = R_1 + R_2$
$R_2' = R_2$

1

	x_1	x_2	s_1	e_2	RHS
z	-1	0	0	2	-8
s_1	0	0	1	$\boxed{2}$	16
x_2	1	1	0	-1	4

1

	x_1	x_2	s_1	e_2	RHS
z	-1	0	-1	0	-24
e_2	0	0	1/2	1	8
x_2	1	1	1/2	0	12

$R_0' = R_0 - R_1$
$R_1' = R_1/2$
$R_2' = R_2 + R_1'$

1

Optimal, as all $\hat{c}_N < 0$ 1

$z^* = -24$
$x_1^* = 0$
$x_2^* = 12$

1

Q2. For the following LP problem **10 marks**

$$\min z = 4x_1 + 2x_2 - x_3$$

$$\text{s.t.} \quad x_1 + 2x_2 \le 6$$

$$x_1 - x_2 + 2x_3 = 8$$

$$x_1,\ x_2 \ge 0,\ \ x_3\ urs$$

State the dual problem, simplify its statement if possible. Provide a brief explanation on how the dual problem has been constructed.

Please provide your solution below:

➢ Asymmetric dual:

apply the following rules:

primal/dual constraint		dual/primal variable
consistent with normal form	⟺	variable ≥ 0
reversed with normal form	⟺	variable ≤ 0
equality constraint	⟺	variable urs

Asymmetric dual:

(P) 1

NOT in normal form

$$\min z = 4x_1 + 2x_2 - x_3$$

$$\text{s.t.} \quad x_1 + 2x_2 \le 6 \quad \text{– reversed with normal form} \rightarrow y_1 \le 0 \quad 1$$

$$x_1 - x_2 + 2x_3 = 8 \quad \text{– "=" constraint} \rightarrow y_2 \text{ - urs} \quad 1$$

$$x_1,\ x_2 \ge 0,\ \ x_3\ urs$$

consistent with normal form → Dual constraints 1 and 2 are in normal form 1 1

1 Dual constraint 3 is "=" constraint

Dual

$$\max\ w = 6y_1 + 8y_2 \quad 1$$

$$\text{s.t.} \quad y_1 + y_2 \le 4 \quad 1$$

$$2y_1 - y_2 \le 2 \quad 1$$

$$2y_2 = -1 \quad 1$$

$$y_1 \le 0,\ \ y_2 \text{ - urs}$$

Use the next page to provide the solution:

Or : Bring (P) into normal form

$$x_3 = x_3' - x_3'' \quad (1)$$

$$\min\ z = 4x_1 + 2x_2 - x_3' + x_3''$$

s.t $-x_1 - 2x_2 \geq -6$ (1)

$x_1 - x_2 + 2x_3' - 2x_3'' \geq 8$

$-x_1 + x_2 - 2x_3' + 2x_3'' \geq -8$ (1)

$x_1\ x_2\ x_3'\ x_3'' \geq 0,$

(1)

(D) symmetrical dual:

$$\max\ -6y_1 + 8y_2 - 8y_3$$

s.t.

$-y_1 + y_2 - y_3 \leq 4$

$-2y_1 - y_2 + y_3 \leq 2$

$2y_2 - 2y_3 \leq -1$

$-2y_2 + 2y_3 \leq 1$

$y_1\ y_2\ y_3 \geq 0$

(1)

symplify (D):

$y_2 - y_3 = \tilde{y}_2$; $\tilde{y}_2 \rightarrow$ urs (1)

$\tilde{y}_1 = -y_1 \rightarrow \tilde{y}_1 \leq 0$ (1)

$2y_2 - 2y_3 = 1$ (1)

Hence :

$$\max\ 6\tilde{y}_1 + 8\tilde{y}_2$$

s.t

$\tilde{y}_1 + \tilde{y}_2 \leq 4$

$2\tilde{y}_1 - \tilde{y}_2 \leq 2$

$2\tilde{y}_2 = -1$

$\tilde{y}_1 \leq 0,\ \tilde{y}_2$ - urs

(1)

Q3. Prove that if A is an $n \times n$ negative definite matrix, then all eigenvalues of A are negative.

Please provide your solution below:

Let λ_i be an eigenvalue of A,

and $x_i \neq 0$ ~ eigenvector:

1 $A x_i = \lambda_i x_i$

As A is negative-def, for any $x \neq 0$

$x^T A x < 0$; Then

1 1

$x_i^T A x_i = x_i^T \lambda_i x_i = \lambda_i x_i^T x_i = \lambda_i |x_i|^2 < 0$

1 1 Hence $\lambda_i < 0$