Table of formulae for Class Test 2

Simplex tableau:

basis	$\mathbf{x_N}$	$\mathbf{x_B}$	rhs
z	$\mathbf{c_B^T B^{-1} N - c_N^T}$	$\mathbf{0}^T$	$\mathbf{c_B^T B^{-1} b}$
$\mathbf{x_B}$	$\mathbf{B^{-1} N}$	$\mathbf{I}$	$\mathbf{B^{-1} b}$

basis	$\mathbf{x_{N_0}}$	$\mathbf{x_{B_0}}$	rhs
z	$\mathbf{c_B^T B^{-1} N_0 - c_{N_0}^T}$	$\mathbf{c_B^T B^{-1} - c_{B_0}^T}$	$\mathbf{c_B^T B^{-1} b}$
$\mathbf{x_B}$	$\mathbf{B^{-1} N_0}$	$\mathbf{B^{-1}}$	$\mathbf{B^{-1} b}$

Primal problem in a normal form	Dual problem in a normal form
$\min\ z = c^T x$ s.t. $Ax \geq b$, (P) $x \geq 0$ $x_j \in x,\ j = 1..n$ - primal variable, associated with jth constraint in the dual problem	$\max\ w = b^T y$ s.t. $A^T y \leq c$, (D) $y \geq 0$ $y_i \in y,\ i = 1..m$ - dual variable, associated with ith constraint in primal problem

Asymmetric dual rules:

primal/dual constraint		dual/primal variable
consistent with normal form	$\Longleftrightarrow$	variable ≥ 0
reversed with normal form	$\Longleftrightarrow$	variable ≤ 0
equality constraint	$\Longleftrightarrow$	variable urs

Strong Duality theorem:

Consider primal LP (P) and the corresponding dual LP (D).

Then $\boldsymbol{c^T x = b^T y}$**,** if and only if x is a primal optimal solution and y is a dual optimal solution.

Complementary slackness theorem:

Let x be a feasible solution to the primal LP (P) and y be a feasible solution to the dual LP (D). Both solutions $x^T = (x_1, x_2, \dots, x_n)$ and $y^T = (y_1, y_2, \dots, y_m)$ are optimal to the primal (P) and dual (D), respectively, if and only if they satisfy

$$\boldsymbol{(c_i - y^T A_i) x_i = 0, i - 1..n} \quad \textit{and} \quad \boldsymbol{y_j (b_j - A_j x) = 0, j = 1 \dots m}$$

Table of formulae and results continues on the next page

Unconstrained non-linear optimisation – some results

- ***Theorem 1.*** Assume that $f(x)$ has continuous second-order partial derivatives for each point $x = (x_1, x_2, \dots, x_n)$. Then $f(x)$ is convex function on D if and only if for each $x \in$ D all principal minors of its Hessian are nonnegative.

- ***Theorem 2.*** Assume that that $f(x)$ has continuous second-order partial derivatives for each point $x = (x_1, x_2, \dots, x_n)$. Then $f(x)$ is concave function on D if and only if for each $x \in$ D and $k = 1 \dots n$ all nonzero k^{th} principal minors of its Hessian matrix have the same sign as $(-1)^k$.

- ***Theorem 3***. If *f(x)* is a convex function and S is a convex set, then any local minimum of the minimisation NLP

$$\min f(x)$$
$$s.t. \;\; x \in S$$

is also a global minimum. If f(x) is a strictly convex function, then the global minimum will be unique.

- ***Theorem 4.*** A symmetric matrix A is positive definite if and only if all its *eigenvalues* are positive. Note: we can also calculate the upper left determinants
- ***Theorem 5***. (Second-order necessary condition) If x^* is a local minimum for an unconstrained NLP problem $min\, f(x)$, then

$\nabla f(x^*) = 0$, and $\nabla^2 f(x^*)$ is positive semidefinite.

- ***Theorem 6***. (Second-order sufficient condition)

If $\nabla f(x^*) = 0$, and $\nabla^2 f(x^*)$ is positive definite,

then x^* is a local minimum for the unconstrained NLP problem $\min f(x)$.

- ***Theorem 7.*** Consider a function $f(x)$ defined in a convex domain. Then

  <u>*Necessary condition for convexity:*</u> *if* $f(x)$ is convex , then $\nabla^2 f(x)$ is positive semidefinite everywhere in its domain.

  <u>*Sufficient condition for strict convexity:*</u> Function $f(x)$ is strictly convex if its Hessian matrix $\nabla^2 f(x)$ is positive definite for all x in its domain.