37242 Introduction to Optimisation Final Examination Spring 2025 Information and instructions - please read ALL the information below carefully:

Information

1. This examination is made available online on Canvas at 12:30 PM (Sydney time) on Wednesday, 12 November 2025. Your completed answers are due by 3:30 PM (Sydney time) on the day of the exam. A pdf file with your *handwritten* solutions must be submitted online via the Assignments tab on Canvas.
2. Any solution that is submitted later than the allowed time will not be accepted for marking.
3. The examination is worth 30% of the marks available in this subject. The contribution each sub-question makes to the total examination mark is indicated in points.
4. This examination is expected to take approximately 2 hours of working time, with an additional 1 hour given for creation of a PDF file and upload. You are advised to allocate your time accordingly. Please allow time to complete the submission process.
5. There are 4 Questions. All Questions are worth equal marks.
6. This examination is an open book examination. The following materials and provisions are permitted:

- Physical and digital recommended textbook
- Course Canvas website
- Any non-programmable calculator

Instructions

1. At the start of the exam download and view the exam paper: 37242-FinalExam-S25
2. *Write* your solutions on your own paper. For each question, write your name, student number, and the question number clearly at the top.
3. After you have completed solutions for all questions, collate them in the order of questions (q1,q2,q3,q4) and scan your answers into a single PDF file, using either Genius Scan or Adobe Scan. Send the PDF to your computer. Make sure that you have included all your working in the file.
4. Please name the file as follows: EXAM_[student number]_[student name]: EXAM_12345678_JohnSmith.pdf
5. Upload the file by clicking on the "Choose a file" button. Make sure that your file has been uploaded to Canvas.
6. Click the "Submit " button . You will have only ONE opportunity to submit.

Notes:

1. You must provide a detailed working out for each question, thus even correct answers without the detailed working out will not be awarded full marks.
2. If you have technical difficulties with your exam, contact the UTS exam hotline (+61 2 9514 3222) for technical support. I will be available during the exam and can be contacted via email: julia.memar@uts.edu.au
3. At the end of the exam (around 3:15PM - 4PM) I will not be able to answer your queries promptly due to a high volume of the questions usually sent at the end of exam.
4. Do not contact me via Canvas during the exam.

Good luck!

With best regards, Julia Memar, 37242 Spring 25 subject coordinator

Examination Conduct

1. You must only attempt this exam once. Any additional attempts should only be used in the event where a serious technical issue has occurred and supporting evidence supporting this will be required.
2. Answer all questions to the best of your ability and perception of the questions' intent, make reasonable assumptions if necessary, to answer all questions.
3. You are not permitted to:
   - obtain assistance by improper means or ask for help from or give help to any other person;
   - take screenshots, record the screen, copy and paste questions or answers or otherwise attempt to take any of the content of this exam out of the exam for any purpose;
   - post any requests for clarification of exam content.

Please note that the examination paper may include features to detect the activities mentioned in p 3.

4. Misconduct action will be taken against you if you breach university rules and /or engage in the activities identified in p.3.

Student declaration

By attempting this exam, I acknowledge that

- I agree to be bound by the university's rules, codes of conduct, and other policies relating to examinations
- I have read and understand the examination conduct requirements for this exam
- I am aware of the university's rules regarding misconduct during examinations
- I am not in possession of, nor do I have access to, any unauthorised material during this examination

In attempting this examination and submitting an answer, candidates are undertaking that the work they submit is a result of their own unaided efforts and that they have not discussed the questions or possible answers with other persons during the examination period. Candidates who are found to have participated in any form of cooperation or collusion or any activity which could amount to academic misconduct in the answering of this examination will have their marks withdrawn and disciplinary action will be initiated.

Question 1 is on the next page

Question 1 – 15 marks

Consider the following linear program:

$$\begin{aligned} \max z = \ & 3x_1 + 5x_2 + 3x_3 \\ s.t. \quad & x_1 + x_2 \leq 5 \\ & x_1 + 2x_3 \leq 12 \\ & x_1 + 2x_2 + 3x_3 \leq 7 \\ & x_1,\ x_2,\ x_3 \geq 0 \end{aligned}$$

The Simplex method ends with the optimal tableau:

basis	x_1	x_2	x_3	s_1	s_2	s_3	RHS
z	0	0	3	1	0	2	19
x_1	1	0	-3	2	0	-1	3
s_2	0	0	5	-2	1	1	9
x_2	0	1	3	-1	0	1	2

where s_1, s_2 and s_3 are the slack variables for the first, second and third constraints, respectively.

a) By using the tableau for the optimal bfs, write down B^{-1}, $B^{-1}N, c_b^T B^{-1}, B^{-1}b$ 4 marks

b) By how much can the right-hand side of the second constraint increase and decrease without changing the optimal basis? 2 marks

c) By how much can the objective coefficient of x_2 increase and decrease without changing the optimal basis? 2 marks

d) By how much can the coefficient of s_3 in the third constraint increase and decrease without changing the optimal basis? 2 marks

e) Assume that another constraint is added to the problem:

$$\begin{aligned} \max z = \ & 3x_1 + 5x_2 + 3x_3 \\ s.t. \quad & x_1 + x_2 \leq 5 \\ & x_1 + 2x_3 \leq 12 \\ & x_1 + 2x_2 + 3x_3 \leq 7 \\ & 3x_1 + x_2 + x_3 \leq 6 \\ & x_1,\ x_2,\ x_3 \geq 0 \end{aligned}$$

By using the tableau above, solve this linear problem 5 marks

Question 2 is on the next page

Question 2 – 15 marks

a) Consider the unconstrained optimisation problem 4 marks

$$\min f(x_1, x_2) = 4x_1^2 + 2x_1 x_2 + 2x_2^2 + x_1 + x_2$$

Use the Steepest descent method to find an approximate solution of the problem, with the initial approximation $x^0 = (0,0)$ and $\varepsilon = \frac{1}{2}$.

b) Consider the unconstrained optimisation problem 6 marks

$$\min f(x_1, x_2) = x_1^4 - 2x_1^2 + 2x_1^2 x_2^2 + x_2^2 - 2x_2$$

Use the Newton's method to find an approximate solution of the problem, with the initial approximation $x^{(0)} = \left(0, \frac{1}{2}\right)$ and $\varepsilon = 0.01$.

c) Consider the following function: 5 marks

$$f(x_1, x_2, x_3) = -2x_1^2 - \frac{x_2^2}{4} + \frac{1}{2}x_2 x_3 - \frac{x_3^2}{2}$$

i. Find all principal minors of its Hessian;

ii. Using the information from part i. or otherwise, determine whether the function is convex, concave or neither.

Question 3 is on the next page

Question 3 - 15 marks

a) Consider the constrained optimisation problem 8 marks

$$\min f(x_1, x_2) = 4x_1^2 + x_2^2 + 3x_1x_2 - x_1 - x_2$$
$$\text{s.t.} \quad 3x_1 + x_2 = 5$$

i. Write the Lagrangian function for this problem.

iii. Use the Lagrangian to find local minimiser(s) for the given problem. Justify your answer.

b) A swimming coach is putting together a relay team for the 400-meter relay. One team member must swim 100 meters of either breaststroke, or backstroke, or butterfly, or freestyle. The coach believes that each swimmer will attain the times given in the table below.

Determine the assignment of swimming style to each team member, so the team's time for the race is minimized. 7 marks

	Time in seconds			
Swimmer	freestyle	breaststroke	butterfly	backstroke
Anne	54	54	51	53
Betty	51	57	52	52
Cloe	50	53	54	56
Dolly	56	54	55	53

Question 4 is on the next page

Question 4 – 15 marks

Consider the pure IP:

$$\begin{aligned} \max z = \quad & -x_1 + 3x_2 \\ s.t. \quad & x_1 + 6x_2 \leq 44 \\ & -x_1 + 2x_2 \leq 8\ 2 \\ & x_1, x_2 \geq 0 \text{ and integer.} \end{aligned}$$

The Simplex procedure for the LP relaxation of the given IP ends with the following final tableau:

basic	x_1	x_2	s_1	s_2	RHS
z	0	0	$\frac{1}{8}$	$\frac{9}{8}$	$\frac{29}{2}$
x_1	1	0	$\frac{1}{4}$	$\frac{-3}{4}$	5
x_2	0	1	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{13}{2}$

where s_1 and s_2 are the slack variables for the first and second constraints, respectively.

Solve the given IP by the cutting-plane algorithm discussed in this subject.

End of the exam paper