Introduction to Optimisation:

Modifications of Simplex Method

Lecture 4

Lecture notes by Dr. Julia Memar and Dr. Hanyu Gu and with an acknowledgement to Dr.FJ Hwang and Dr.Van Ha Do

Example 2 – solve with Big M method

- $min\, z = 2x_1 + 3x_2 +$

s.t.

$$\frac{1}{2}x_1 + \frac{1}{4}x_2 = 4$$

$$x_1 + 3x_2 = 20$$

$$x_1 + x_2 = 10$$

$$x_1,\ x_2, \geq 0$$

Example 2 – solve with Big M method

Two-phase Simplex method

Phase I .

1. Make sure all $rhs \geq 0$;
2. Add slack/subtract excess variables to make equality constraints;
3. For each constraint i without a slack add artificial variable a_i;
4. Solve $\min w = \sum a_i$ with the modified constraints from step 3.

Two-phase Simplex method

Phase II

- If $w_{opt} > 0$, then (I) is ________________________________

- If $w_{opt} = 0$ and there are no a_i in x_B then

  1. ________________________
  2. ________________________
  3. ________________________

- If $w_{opt} = 0$ and there are a_i in x_B then

  1. ________________________
  2. ________________________
  3. ________________________

Example 3 (Solve with two-phase Simplex)

- $\min z = 2x_1 + 3x_2$

s.t. $\frac{1}{2}x_1 + \frac{1}{4}x_2 \leq 4$

$x_1 + 3x_2 \geq 20$

$x_1 + x_2 = 10$

$x_1, \ x_2 \geq 0$

Example 3

Example 4 (Solve with two-phase Simplex)

- $\max z = 4x_1 + 5x_2$

$$\text{s.t.} \qquad 2x_1 + 3x_2 \leq 6$$

$$3x_1 + x_2 \geq 3$$

$$x_1,\ x_2 \geq 0$$

Example 4

Revised Simplex method: Introduction

- Large-scale LP problems contain large amounts of data, and
- Require even larger numbers of computations in each Simplex iteration
- Consider $\min\ (\boldsymbol{or\ max})\ z = \boldsymbol{c^T x}$

$$s.t. \quad \boldsymbol{Ax = b}, \quad \boldsymbol{x \geq 0},$$

For the chosen $\boldsymbol{x_B}$, A, B, c^T, c_B^T can be obtained by

After B^{-1} is calculated, we can calculate the tableau:

basis	$\mathbf{x}$	rhs
z	$\mathbf{c}_B^T \mathbf{B}^{-1}\mathbf{A} - \mathbf{c}^T = \hat{\mathbf{c}}^T$	$\mathbf{c}_B^T \mathbf{B}^{-1}\mathbf{b}$
$\mathbf{x}_B$	$\mathbf{B}^{-1}\mathbf{A} = \hat{\mathbf{A}}$	$\mathbf{B}^{-1}\mathbf{b} = \hat{\mathbf{b}}$

Revised Simplex method: Introduction

- Decomposed tableau for $x^T = (x_N | x_B)$:

basis	$\mathbf{x_N}$	$\mathbf{x_B}$	rhs
z	$\mathbf{c_B^T B^{-1} N - c_N^T}$	$\mathbf{0}^T$	$\mathbf{c_B^T B^{-1} b}$
$\mathbf{x_B}$	$\mathbf{B^{-1} N}$	$\mathbf{I}$	$\mathbf{B^{-1} b}$

- For initial x_{B_0} $B_0 =$ and $A =$ and $B_0^{-1} =$
- The initial tableau :

basis	x_{N_0}	x_{B_0}	rhs
z			
x_{B_0}			

Revised Simplex method: Introduction

For the next x_{B_1}

- $B_1^{-1}(N_0|I) =$

- $c_{B_1}^T B_1^{-1} A - c^T =$

basis	x_{N_0}	x_{B_0}	rhs
z			
x_{B_1}			$B_1^{-1} b$

Revised Simplex method: Introduction

- Observations:

Revised Simplex method: Introduction

If $(x_{N_1}|x_{B_1})$ is not optimal, select the next x_{B_2}

- $B_2^{-1}(N_0|I) =$

- $c_{B_2}^T B_2^{-1}A - c^T =$

basis	x_{N_0}	x_{B_0}	rhs
z			
x_{B_2}			$B_2^{-1}b$