Problems for computer labs 2-3

Introduction to Optimisation

Modeling: a summary

The steps involved in the formulation of a mathematical optimisation model are:

a. Identify the decision variables (whose quantities can be controlled by the decision maker).

b. Construct the objective function and constraints (restrictions) in terms of the decision variables. (You should not introduce new variables (unknowns) at this point.

c. Ask yourself whether there is any hidden assumption that may exist,

e.g. integer decision variables, fractional decision variables.
The most common “hidden” assumption is that the variables are nonnegative.

d. Write the complete formulation as a linear program, objective first, followed underneath by the constraints.

e. It is usually a good idea to write the formulation out first before constructing the spreadsheet until you get experience at formulation and structuring spreadsheets.

Blending problem 1

A one-kilogram pack of dog food must contain protein (at least 31%), carbohydrate (at least 38%), and fat (between 13% and 15%, that is greater than or equal to 13% and less than or equal to 15%). Three foods (food1, food2 and food3) are to be blended together in various proportions to produce a least-cost pack of dog food satisfying these requirements. The information for one kilogram of each food is given in the table below.

	protein (*grams*)	*carbohydrate* (*grams*)	*fat* (*grams*)	*price* (*dollars*)
food1	300	500	90	3
food2	450	300	160	1
food3	200	400	200	2

Blending problem 2

A company should produce steel from the following three alloys:

	cost (per tonne)	carbon	chrome	silicon	available (tonne)
Alloy 1	\$52	5%	0.1%	3%	unlimited
Alloy 2	\$71	3.8%	0.4%	2%	2000
Alloy 3	\$46	2%	0.3%	2.8%	5000

The chemical content of a blend is the weighted average of the chemical content of its components. For example, the portion of carbon in a blend of 20 tonnes of Alloy 1 and 30 tonnes of Alloy 2 is

$$\frac{20 \times 0.05 + 30 \times 0.038}{20 + 30} = 0.0428, \quad \text{that is } 4.28\%.$$

The steel should satisfy the following quality requirements

	at least	not more than
carbon	3.5%	4 %
chrome	0.2%	0.5 %
silicon	2.7%	2.7 %

and will be sold for \$65 per tonne.

The company wishes to maximise its profit.

Production planning

Giapetto's wooden soldiers and trains. Each soldier sells for $27, uses $10 of raw materials and takes $14 of labor & overhead costs. Each train sells for $21, uses $9 of raw materials, and takes $10 of overhead costs. Each soldier needs 2 hours finishing and 1 hour carpentry; each train needs 1 hour finishing and 1 hour carpentry. Raw materials are unlimited, but only 100 hours of finishing and 80 hours of carpentry are available each week. Demand for trains is unlimited; but at most 40 soldiers can be sold each week. How many of each toy should be made each week to maximize profits?

(Winston 3.1, p. 49)

Integer programming modelling: Portfolio selection problem

Over a 3-year planning horizon, an investor can invest in the following projects:

project	investment			return
	Year 1	Year 2	Year 3	
Project 1	2	8	4	35
Project 2	5	4	10	22
Project 3	9	3	13	39
Project 4	1	5	7	30
Project 5	11	6	3	31
Project 6	4	4	6	19
Project 7	2	12	4	20

The following funds are available over this 3-year planning horizon:

	Year 1	Year 2	Year 3
available funds	33	40	41

Portfolio selection problem

The investment strategy should satisfy the following requirements:

- the investor can invest in at most four projects;
- if the investor invests in Project 1, then he can invest in at most two of the five projects - Project 2, Project 3, Project 4, Project 5 and Project 7;
- if the investor invests in Project 2, then he must also invest in at least two of the three projects - Project 3, Project 4 and Project 6;
- if the investor invests in either Project 5 or Project 6 or in both, then he must invest in at least one of the two projects - Project 2 or Project 4.
- if the investor invests in Project 1, then he cannot invest in Project 6;
- if the investor invests in one of the two projects - Project 1 or Project 3, then he must also invest in the other one;

The investor wishes to maximise the total return from his investments.

Integer programming modelling: workforce planning

A company has only full-time employees. The following table specifies the number of personnel required each day:

	required number
Monday	20
Tuesday	13
Wednesday	10
Thursday	12
Friday	15
Saturday	9
Sunday	17

If each employee works five days per week, what is the minimum number of employees needed in order to meet the daily requirements?