Extra problem (From Winston 2009): A company has idle funds of $20 million available for investment in short-term and long-term securities. Government regulation require that no more than 80% of all investment be in long-term securities, and no more than 40% in short-term securities, and the ratio of long-term to short-term investments not exceed 3 to 1. Long term investments currently yield 15% pa while short-term investments yield 10 %. Solve this problem graphically.

1. DV:

$LT \rightarrow$ amount invested in long-term,

ST — amount invested in short-term

2. OF: max $1.15 LT + 1.1 ST$

s.t.

$$LT + ST \leq 20$$

$$LT \leq 0.8 \times (LT + ST)$$

$$ST \leq 0.4 (LT + ST)$$

$$LT \leq 3 ST$$

$$LT, ST \geq 0.$$

is there a redundant constraint?

→ Simplify the constraints first:

s.L.

$LT + ST \leq 20$

$LT \leq 0.8 \times (LT + ST) \rightarrow$ $0.2 LT \leq 0.8 ST$

                                 $LT \leq 4 ST$

$ST \leq 0.4 (LT + ST) \rightarrow$ $0.6 ST \leq 0.4 LT$

$LT \leq 3 ST$! $\frac{3}{2} ST \leq LT$

Hence the constraints are:
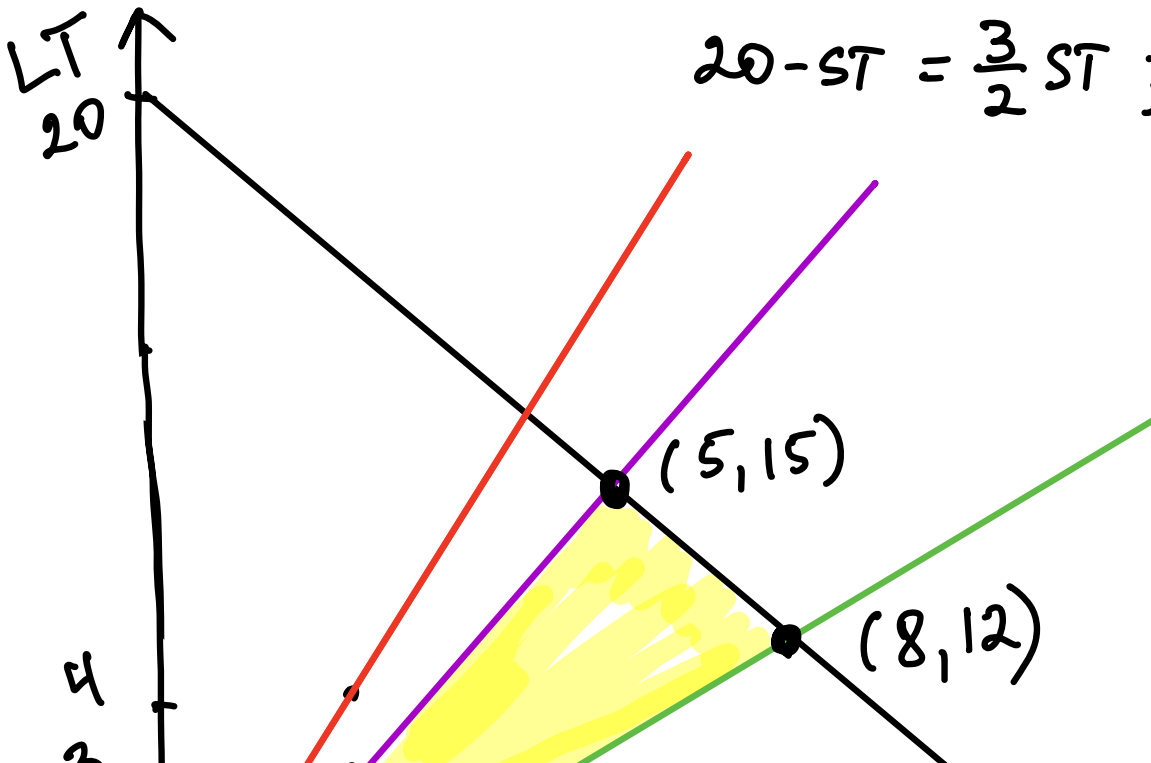
- $LT \leq 20 - ST$
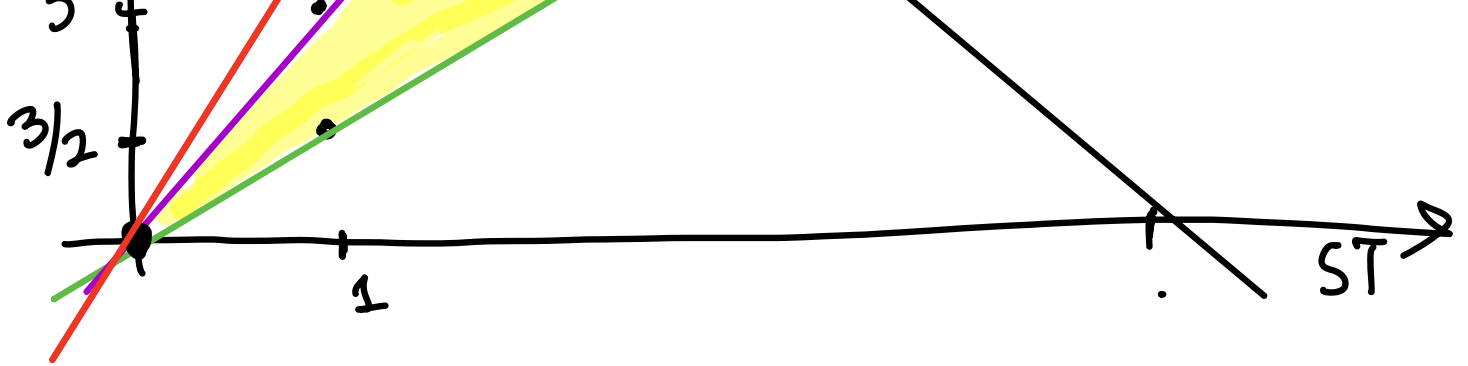- $LT \leq 4 St \longrightarrow$ This constraint is redundant.
- $LT \geq \frac{3}{2} ST$
- $LT \leq 3 ST$

$20 - ST = 3 ST \; ; \; ST = 5 \atop LT = 15$

$20 - ST = \frac{3}{2} ST \; ; \; ST = 8 \atop LT = 12$

(5,15)

(8,12)

LT
20

4

$$z = 1.15 \, LT + 1.1 \, ST$$

$$z(8, 12) = 22.6 \, \$mln$$

$$z(5, 15) = 22.75 \, \$mln \rightarrow maximum$$