37242 Introduction to Optimisation

Tutorial 1

For each of the following problems:

a. Draw the feasible region.
b. What are the extreme points ?
c. Solve the problem, or determine that solution does not exist

1) $max\ 5x_1 + 4x_2$

$$s.t. \quad 3x_1 + 2x_2 \leq 120$$
$$x_1 + x_2 \leq 50$$
$$x_1, x_2 \geq 0$$

2) $max\ 5x_1 + 4x_2$

$$s.t. \quad 3x_1 + 2x_2 \leq -20$$
$$x_1 + x_2 \leq 50$$
$$x_1, x_2 \geq 0$$

3) $max\ 5x_1 + 4x_2$

$$s.t. \quad 3x_1 + 2x_2 \geq 120$$
$$x_1 + x_2 \geq 50$$
$$x_1, x_2 \geq 0$$

4) $min\ 5x_1 + 4x_2$

$$s.t. \quad 3x_1 + 2x_2 \geq 120$$
$$x_1 + x_2 \geq 50$$
$$x_1, x_2 \geq 0$$

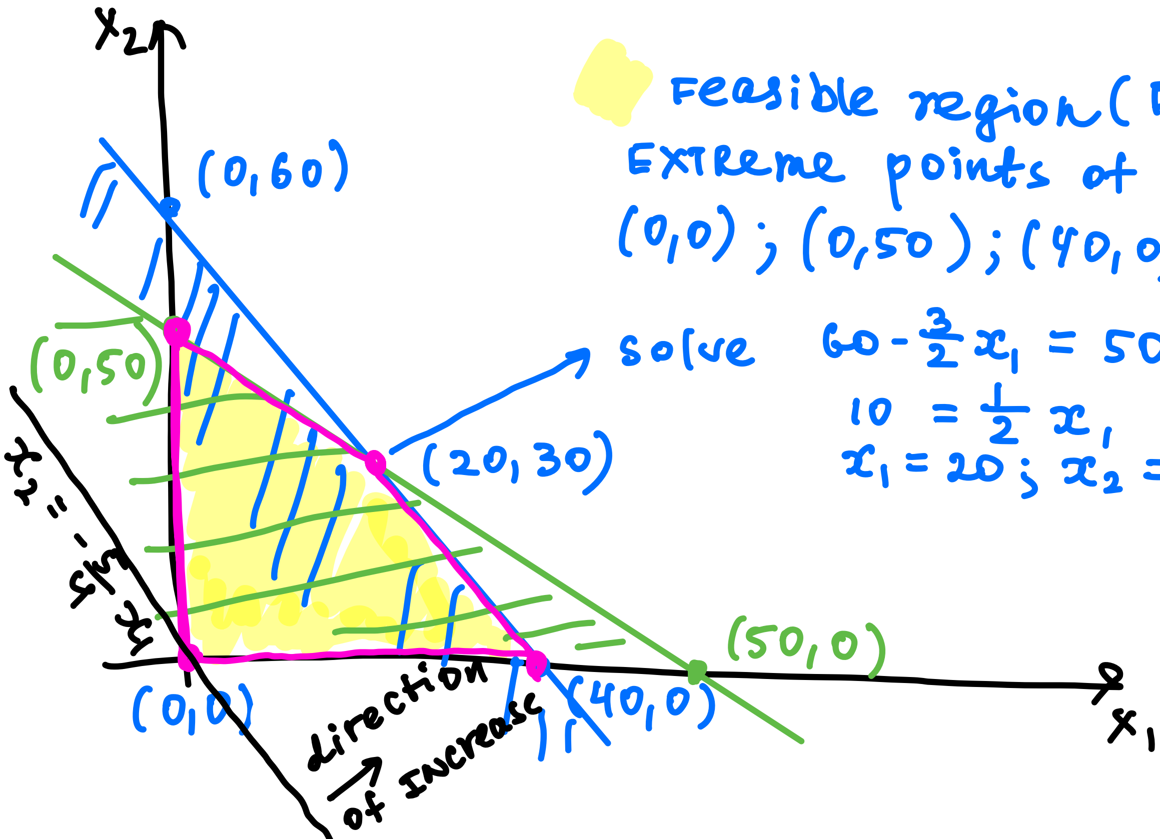
- Is the optimal solution unique?

1) $\max 5x_1 + 4x_2$

$$\text{s.t.} \quad 3x_1 + 2x_2 \le 120$$
$$x_1 + x_2 \le 50$$
$$x_1, x_2 \ge 0$$

$\rightarrow$ $3x_1 + 2x_2 = 120$

- $x_2 = 60 - \frac{3}{2}x_1$
- $x_2 = 50 - x_1$



Feasible region (FR)

Extreme points of FR:

$(0,0)$; $(0,50)$; $(40,0)$; $(20;30)$

solve $60 - \frac{3}{2}x_1 = 50 - x_1$

$10 = \frac{1}{2}x_1$

$x_1 = 20$; $x_2 = 30$

$z = 5x_1 + 4x_2 \leftarrow$ OF

Isolines of OF : $5x_1 + 4x_2 = C$ $\qquad D = \frac{C}{4}$

$x_2 = \underline{\underline{D}} - \frac{5}{4}x_1$

$z(0,0) = 0$

$z(0,50) = 200$

$z(40,0) = 200$

$z(20,30) = 220 \rightarrow$ max $\rightarrow z_{max} = 220$

$x = \begin{pmatrix} 20 \\ 30 \end{pmatrix}$ is optimal solution

2) $\max 5x_1 + 4x_2$

s.t. $3x_1 + 2x_2 \leq -20$

$x_1 + x_2 \leq 50$

$x_1, x_2 \geq 0$

as both x_1 and x_2 are non-neg-ve

$3x_1 + 2x_2 \geq 0$

For any $x_1, x_2 \geq 0$

$\downarrow$

1st constraint can not be satisfied

$3x_1 + 2x_2 = -20$

$\downarrow$

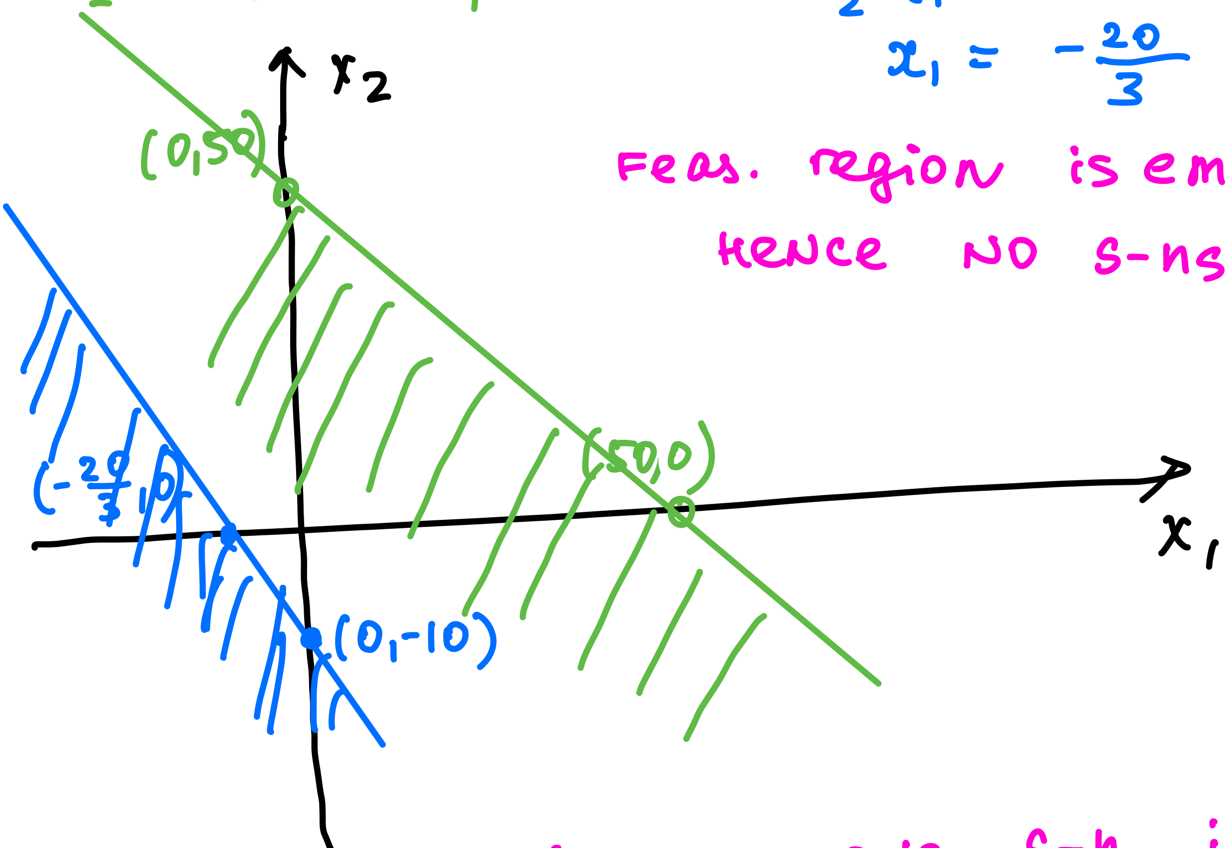
$x_2 = -10 - \frac{3}{2} x_1$

$-10 - \frac{3}{2} x_1 = 0$

$\frac{3}{2} x_1 = -10$

$x_1 = -\frac{20}{3}$

$x_2 = 50 - x_1$



Feas. region is emty, hence no s-ns

Q: Will the problem have s-n if non-negativity constraint is removed?

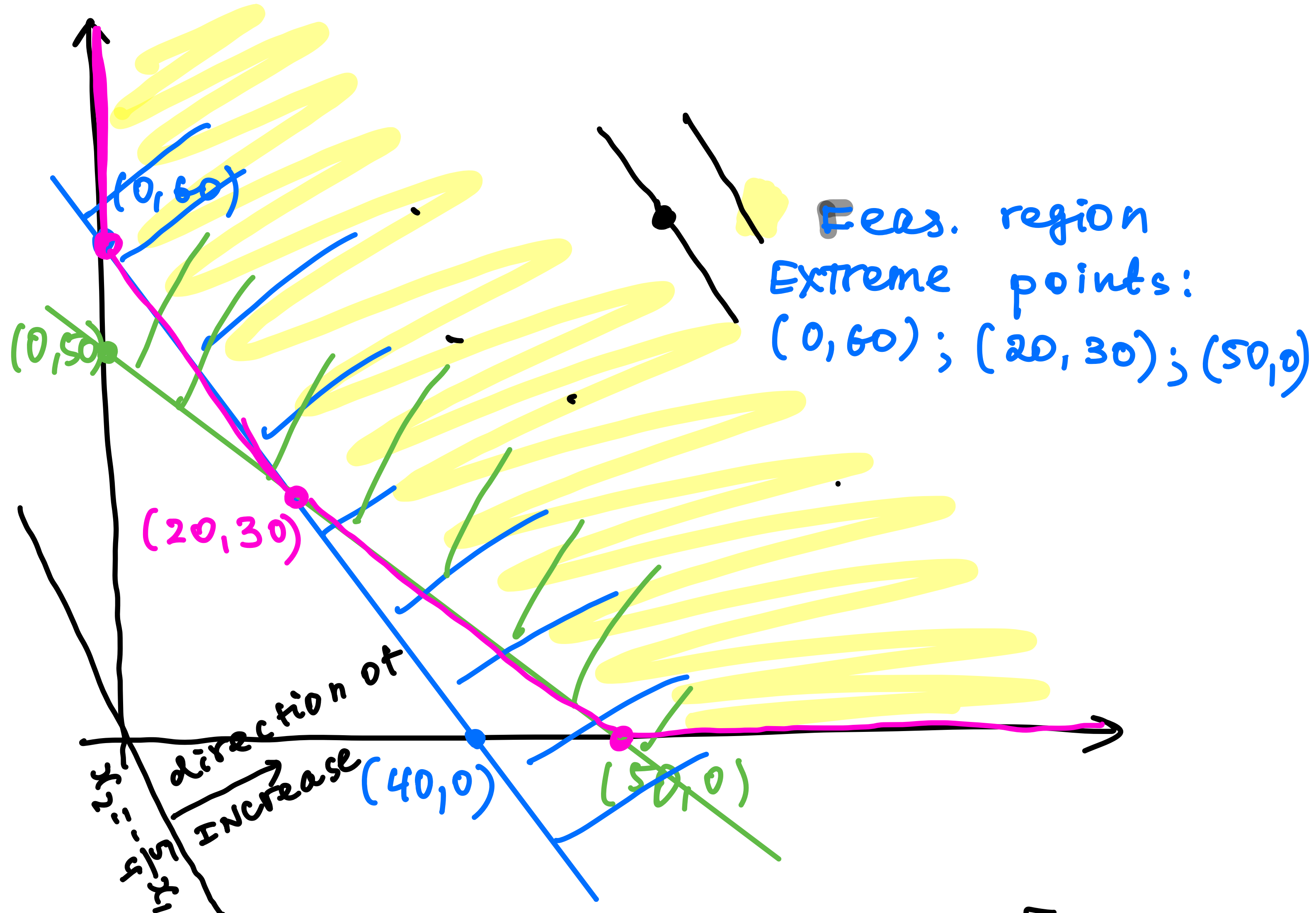
$$x_1, x_2 \geq 0$$

3) $max\ 5x_1 + 4x_2$

s.t. $\quad 3x_1 + 2x_2 \geq 120$

$$x_1 + x_2 \geq 50$$

$$x_1, x_2 \geq 0$$



$z = 5x_1 + 4x_2$; Isolines : $x_2 = D - \frac{5}{4}x_1$

As direction of Improvement of OF coinsides with direction of unboundedness of feas. region, there is no opt. s-n → problem is unbounded

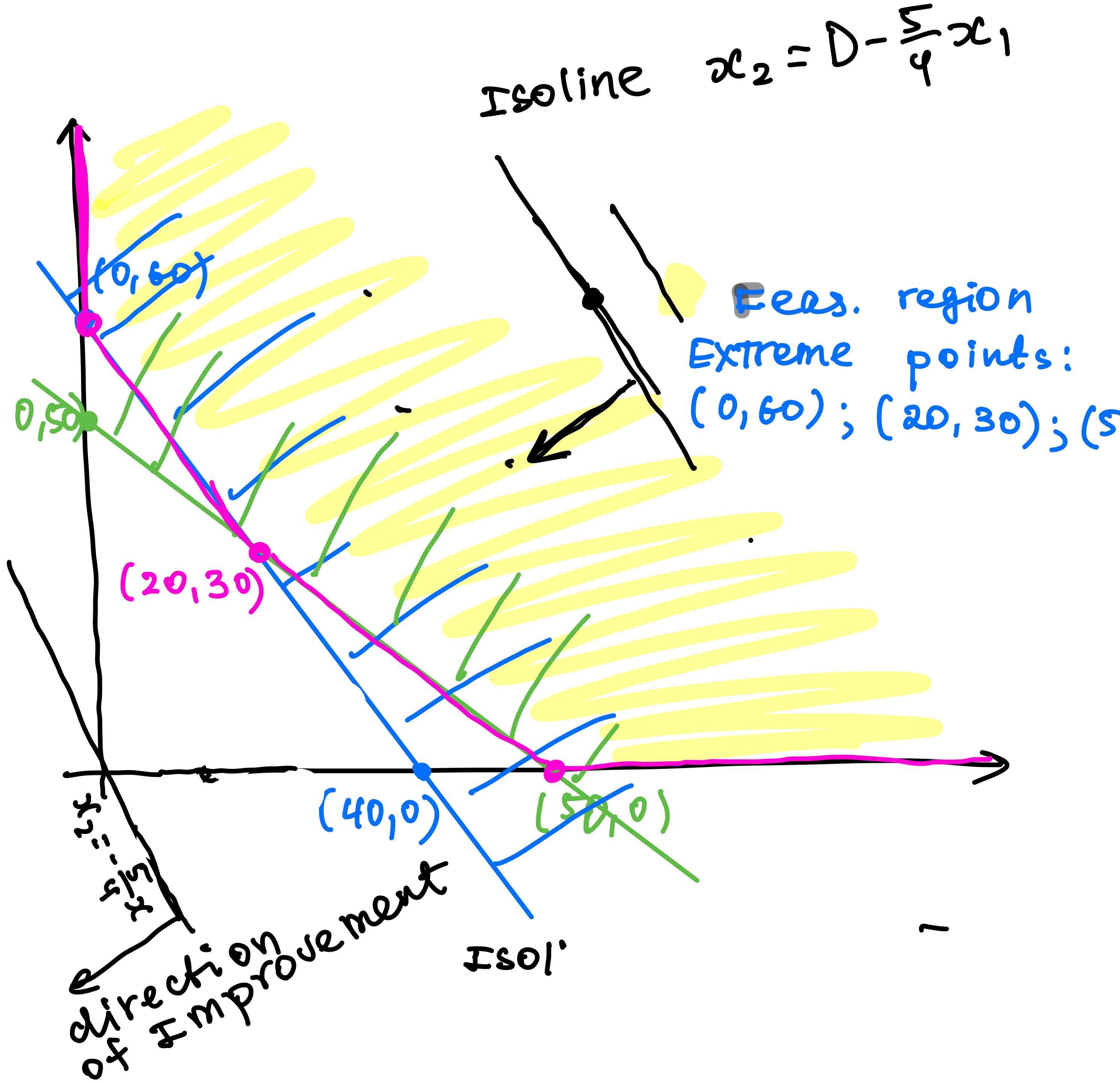
4) $\min 5x_1 + 4x_2$

s.t. $3x_1 + 2x_2 \geq 120$

$x_1 + x_2 \geq 50$

$x_1, x_2 \geq 0$

- Is the optimal solution unique?



$z(0,60) = 240$

$z(20,30) = 220 \rightarrow z_{min} = 220$ and

$z(50,0) = 250$ $\quad x = \begin{pmatrix} 20 \\ 30 \end{pmatrix}$ is optimal s-n

As direction of Feas. region unboundedness is opposite to direction of Improvement of OF, optimal s-n exists.

Extra problem (From Winston 2009): A company has idle funds of \$20 million available for investment in short-term and long-term securities. Government regulation require that no more than 80% of all investment be in long-term securities, and no more than 40% in short-term securities, and the ratio of long-term to short-term investments not exceed 3 to 1. Long term investments currently yield 15% pa while short-term investments yield 10 %. Solve this problem graphically.

1. DV: Let LT be amount invested (in \$ mln) in long-term
   ST be amount invested (in \$ mln) in short term

2. OF: max $1.15\,LT + 1.1\,ST$

s.t.

$$LT + ST \leq 20$$

$$LT \leq 0.8\,(LT + ST)$$

$$ST \leq 0.4\,(LT + ST)$$

$$\frac{LT}{ST} \leq \frac{3}{1} \rightarrow LT \leq 3ST$$

$$LT, ST \geq 0.$$

Is there a redundant constraint?

Solve graphically.