37242 Introduction to Optimisation

Tutorial 4

1. Consider the LP in standard form:

$$
\begin{array}{rrcrcrcccccl}
\max z = & x_1 & + & x_2 & + & x_3 & & & & & & \\
s.t. & x_1 & + & 2x_2 & + & 3x_3 & + & s_1 & & & = & 5 \\
 & 2x_1 & + & 4x_2 & + & x_3 & & & + s_2 & & = & 6 \\
 & x_1 & + & 3x_2 & + & 3x_3 & & & & + s_3 & = & 6 \\
 & \multicolumn{11}{l}{x_1, x_2, x_3, s_1, s_2, s_3 \geq 0.}
\end{array}
$$

   (a) If the basis $\mathbf{x}_B$ is (s_1, s_2, s_3), what are the values for $\mathbf{x}_B$ and $\mathbf{x}_N$, and what is the objective value?

   (b) If the basis $\mathbf{x}_B$ is (x_1, x_2, x_3), what are the values for $\mathbf{x}_B$ and $\mathbf{x}_N$, and what is the objective value?

   (c) If the basis $\mathbf{x}_B$ is (x_1, x_2, s_1), what are the values for $\mathbf{x}_B$ and $\mathbf{x}_N$. Is this a basic feasible solution (bfs)?

2. Solve the following LP with Simplex method in the tabular form:

$$
\begin{array}{rrcrl}
\min z = & -x_1 & - & 2x_2 & \\
s.t. & x_1 & + & x_2 & \leq 2 \\
 & 2x_1 & + & x_2 & \leq 1
\end{array}
$$

$$
\begin{array}{rrcrcl}
\max & 2x_1 & + & x_2 & & \\
\text{s.t.} & 7x_1 & + & 4x_2 & \leq & 14 \\
 & x_1 & & & \leq & 2 \\
 & \multicolumn{5}{l}{x_1, x_2 \geq 0.}
\end{array}
$$

3. Show that the set of all $\mathbf{x}$ satisfying

$$\mathbf{Ax} \leq \mathrm{b}$$
$$\mathbf{x} \geq \mathbf{0}$$

is convex. (Hint: take two arbitrary points satisfying the constraints, and consider a convex combination of these points)