Tutorial 7

1. If we solve the LP

$$\begin{array}{rrcrcrcr} \min z = & -8x_1 & - & 3x_2 & + & x_3 & & \\ \text{s.t.} & x_1 & + & 2x_2 & - & x_3 & \leq & 4 \\ & 2x_1 & + & 5x_2 & & & \leq & 16 \\ & 4x_1 & & & + & x_3 & \leq & 12 \\ & \multicolumn{7}{l}{x_1, x_2, x_3 \geq 0,} \end{array}$$

then the Simplex method ends with the optimal solution, and the final tableau (adding slack variables s_1, s_2, s_3 to the three constraints) is:

basic	x_1	x_2	x_3	s_1	s_2	s_3	rhs
z	0	0	$-\frac{9}{8}$	$-\frac{3}{2}$	0	$-\frac{13}{8}$	$-25\frac{1}{2}$
x_2	0	1	$-\frac{5}{8}$	$\frac{1}{2}$	0	$-\frac{1}{8}$	$\frac{1}{2}$
s_2	0	0	$\frac{21}{8}$	$-\frac{5}{2}$	1	$\frac{1}{8}$	$\frac{15}{2}$
x_1	1	0	$\frac{1}{4}$	0	0	$\frac{1}{4}$	3

(a) By how much can the right-hand side of the first constraint increase and decrease without changing the optimal basis?

(b) By how much can the objective coefficient of x_3 increase and decrease without changing the optimal basis?

(c) By how much can the objective coefficient of x_1 increase and decrease without changing the optimal basis?

(d) By how much can the coefficient of x_3 in the first constraint increase and decrease without changing the optimal basis?

2. Consider the LP:

$$
\begin{array}{rrrrrrrrrrr}
\min z = & -3x_1 & + & x_2 & - & 7x_3 & & & & & \\
\text{s.t.} & x_1 & + & 4x_2 & + & 2x_3 & + & x_4 & & = & 15 \\
& 3x_1 & - & 2x_2 & + & 4x_3 & & & + \; x_5 & = & 5 \\
& \multicolumn{10}{l}{x_1, x_2, x_3, x_4, x_5 \geq 0.}
\end{array}
$$

The Simplex method ends with the optimal solution, and the final tableau is:

basic	x_1	x_2	x_3	x_4	x_5	rhs
z	-2	0	0	-0.5	-1.5	-15
x_2	-0.1	1	0	0.2	-0.1	2.5
x_3	0.7	0	1	0.1	0.2	2.5

(a) By how much can the objective coefficient of x_3 increase and decrease without changing the optimal basis?

(b) By how much can the coefficient of x_1 in the first constraint increase and decrease without changing the optimal basis?

(c) Suppose the right-hand side of the original problem is replaced by $\mathbf{b}' = \begin{bmatrix} 15 \\ 5 + \Delta \end{bmatrix}$.

i. What values can Δ take on without changing the optimal basis?

ii. What is the optimal objective value (as a function of Δ) over the range found in part i. ?

(d) Solve the following LP:

$$
\begin{array}{rrrrrrrrrrr}
\min & -3x_1 & + & x_2 & - & 7x_3 & & & & & \\
\text{s.t.} & x_1 & + & 4x_2 & + & 2x_3 & + & x_4 & & = & 16 \\
& 3x_1 & - & 2x_2 & + & 4x_3 & & & + \; x_5 & = & 5 \\
& \multicolumn{10}{l}{x_1, x_2, x_3, x_4, x_5 \geq 0.}
\end{array}
$$

(e) Solve the following LP:

$$
\begin{array}{rrrrrrrrrrr}
\min & -3x_1 & + & x_2 & - & 7x_3 & & & & & \\
\text{s.t.} & x_1 & + & 4x_2 & + & 2x_3 & + & x_4 & & = & 15 \\
& 3x_1 & - & 2x_2 & + & 4x_3 & & & + \; x_5 & = & 6 \\
& \multicolumn{10}{l}{x_1, x_2, x_3, x_4, x_5 \geq 0.}
\end{array}
$$

(f) Compare the objective value in parts d) and e) to those in the initial problem, and compare the changes to the components in the vector $\mathbf{c_B}^T\mathbf{B}^{-1}$.