32931 - Individual Learning Plan

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March 15, 2026

1   Introduction

Analogous to the commercial deployment of artificial intelligence, radio technology was considered a widespread revolution in engineering, being comercially employed in broadcasting, radar, and later bluetooth, Wi-Fi. Consequently, radio quickly became a popular topic of study among electrical engineers. Maxwell's equations paired with differential equations derived from circuit theory can be used to explain much of the physical behaviour of waves, while Fourier analysis offers the tools that deal with signal processing through radio waves. Ideally, engineers would like to deal with a purely mathematical framework that reflects the known corpus of results regarding radio. Futhermore, such a theory should be well tested to verify that the extent of questions that mathematics can resolve is broad; the proposed project aims to create such a theory and demonstrate its effectiveness.

Provide the background for the research question, gaps and objectives that the project acknowledges. Then this document will discuss the chosen preparatory courses that will be undertaken to satisfy the research question, fill research gaps, and succesfully fulfil the project's objectives.

2   The Research Task

The project begins with an expository goal; to demonstrate the role of Fourier analysis within the field of signal processing and radio engineering. It will discuss the state of Fourier Analysis as a mathematical discipline and demonstrate how it can be used as a formal representation for signals and radio waves.

Once this theory had been developed, it will be used to answer various open problems and conjectures proposed by Nahin (2025) in 'The Mathematical radio'. Rather than experimentally verifying a hypothesis to answer open problems and conjectures, the project takes the route of generating a formal proof within a mathematical framework that demonstrates radio.

This section's objective is to state the research question, gaps, objectives, methodology and methods that will comprise the basis of this project.

2.1 Research gaps

The research gaps this project attempts to address are in large part due to a recent treatise of mathematical radio, 'The mathematical radio: Inside the magic of AM, FM, and signle-sidebanded'. This book will be discussed in further detail in a later section as it is the central inspiration for this project, however its research gaps are in the 'challenge questions' that it poses. The book discusses application of mathematics for radio technology, applying techniques to offer solutions, partial solutions, and possible proof methods regarding a variety of problems ion mathematical radio. The questions with partial or absent solutions will form a part of the research gaps in this project.

Additionally, the book lacks more specific applications of mathematics to antenna theory, with models being assumed rather than derived as they require "a (much) more analytical study of antennas than we are doing here [in Nahin (2025)]". The project also to seeks to extend the mathematical methods in Nahin (2025) to resolve problems regarding the optimision of antenna design.

2.2 Research question

Given that the project consists of both an expository and problem-resolving component, the project consists of two research questions that define the direction of each respective component.

- Expository question: *How is mathematics (specifically Fourier analysis) used as a mathematical basis for radio technology and signal processing?*

- Problem-resolving question: *Can this mathematics alone be used to answer the open questions in radio technology proposed by Nahin (2025)?*

The expository question deals with introducing the mathematics associated with radio technology, and then demonstrating how problems in radio engineering can be formalized into purely mathematical terms where these tools are effective. This would require translating radio engineering and electromagnetic concepts into a formal framework rather than an empirical one.

On the other hand, the problem-resolving question is a followup to the expository question which seeks to demonstrate the effectiveness of our 'mathematical basis'. Mathematics is notorious for its 'unreasonable effectiveness' in physical and engineering matters, and this project will reaffirm mathematics' role in radio engineering by using it as a framework to answer the problems portrayed in Nahin (2025) as well as more advanced problems that naturally arise in the study of antennae.

2.3 Research objectives

The research question prompts six different research objectives whose resolution will determine the success of this project.

Research objectives grow directly from research questions and their related gaps; therefore this document will naturally derive the objectives from each question. The

- State the fundamentals of Fourier analysis and contrast how the Lebesgue integral supplements this theory.

- Describe how this mathematics can be used to axiomatise the study of radio technology.

These objectives are clearly in reference to developing a treatise on mathematical radio, discussing the prerequisite mathematics and a demonstration of their relevance to radio technology.

The problen-resolving question can be further subdivided into the following research objectives.

- Translate the open problems and conjectures proposed by Nahin (2025) into first-order logical statements.

- Generate proofs for various open problems and conjectures proposed by Nahin (2025).

- Propose and attempt reolutions for futher problems in mathematical radio relating to antenna theory

- Assess the efficacy of a mathematical framework for studying radio technology.

The objectives essentially define a linear process for answering the questions of mathematical radio and structuring the second part of the project; find a suitable way to translate questions into mathematics, solve them, solve further questions, and then reflect on the fluidity of this process.

It is also noteworthy that the research objectives derived from the expository question become necessities for the subsequent objectives; one is required to build (or have accessible) an adequate theory of mathematics before beginning to attempting to answer such technical questions in radio engineering.

2.4 Research methodology

Mathematics is a theoretical discipline whose correctness operates on the basis of a formalist approach rather than experimental verification, therefore research methodologies based on experiments, statistical inference, and surveying are incompatible with the philosophy of the project. Indeed, the project will work entirely within the framework of using previous theorems to propound results regarding radio technology and signal processing. For this reason, the thesis is a *non-empirical methodology*, rather than the classic quantitative and qualitative methodologies.

The project will work within the modern state of Fourier analysis and assume Maxwell's equations as further axioms regarding electric and magnetic fields. Indeed, the first part of the project will be dedicated to the development of such a formal mathematical system based on previously experimented and accepted scientific knowledge about radio.

2.5   Research methods

The main challenge of the project will be answering the open problems of Nahin, of which sophisiticated research methods may be required to synthesise proofs. Despite working within the non-empirical research methodology, one can use a more liberal class methods to inspire a formal proof. Though empirical methods cannot substitute a formal proof, they may suggest the correct intuition and aid such a derivation as a 'heuristic' method.

2.5.1   Quantitative simulation (a quantitative method)

To gain an understanding for a mathematical object, numerical methods, simulations, and graphing software (Sage, Wolfram, MATLAB etc.), may potentially be leveraged as methods to gain intuition on certain properties of an object that could be useful. This is perhaps the most famous non-formal heuristic method employed by mathematicians to inspire proofs.
For instance, by plotting a numerical solution to a differetial equation, even if an analytical solution is unknown, the behaviour of the plot may suggest that the function has properties such as periodicity, being bounded, vanishing etc. This is an example of how a quantitative method may suggest qualitative properties. It is important to reiterate that the project does not constitute this as a formal verification; it is merely a tool to inspire proofs. From a synthetic point of view, if one has a strong intuition for some property but cannot prove it, a mathematian may concede and make the property apart of a sufficient condition in subsequent theorems, though proving the original property would however become a new research gap for a future project.

2.5.2   Deriving lemmas (a non-empirical method)

Perhaps a certain calculation or certain fact become a recurring theme in the answering of open questions. In this case it may be convenient for the mathematician to state a lemma that becomes a stepping stone to mechanise proofs to become more elegant, less abstruse, and readable. As opposed to quantitative simulations like numerically solving differential equations, lemmas are formally derived and constitute as partial solutions to open problems themselves.

3   Literature and Coursework

To undertake such a project, one must be familiar with the current state of research invested into the mathematics of radio. This requires reading related literature on mathematical interpretations of radio and being educated through courses on the mathematical techniques employed.

Literature and coursework establishes a positive feedback loop with the research question; the literature provides the research gaps to motivate the research questions and project, while the research questions suggest developing a background in the field by exploring related courses and literature.

3.1 Coursework

Coursework will be quintessential in understanding the status quo of mathematical radio such that the research questions may be appropriately addressed.
With the exception of the TRM module, the courses will exhibit a 'chain dependency'; the modules flow from mathematical, to physical, to engineering-related. This is done intentionally so that the mathematics studied at the beginning of the 'chain' can be used to reduce problems of a physical nature into pure mathematics, and similarly for enginering.

3.2 TRM compulsory module (1cp)

This module seeks to equip researchers with the skills to thoroughly devise a research plan by choosing a methodology, underlining research gaps, employing research method to conduct an experiment and apply an appropriate metric to evaluate the experiment's result.
Although much of the research skills are specifically tailored to computer scientists who are experimental models (for example, experiments on specific-oriented neural networks) or conducting surveys (for instance, surverys regarding human interaction with AI interfaces), the general framework gives hints on how to seek out research method techniques that are more faithful to the non-empirical methodology. Therefore even if its contents is not entirely applicable to the project in question, the transdiciplinary concepts such as research gaps and more general theory of research methodologies and methods will determine the strucutre of the project.

3.3 Digital signal processing; Fundamentals (3cp)

https://www.coursera.org/learn/dsp1
The project requires a source for the mathematical theory of Fourier analysis on which to base its treatise. Furthermore, methods from Fourier analysis will become vital elements in constructing proofs for the open questions of Nahin (2025). Because of this fundamental requirement, a course that lays the rudiments of signal processing within a Fourier analysis framework proves invaluable.
The course provides information on the theory of Fourier analysis and discrete Fourier transform specifically for applications in signal processing, providing essential mathematical techniques that will synthesise a robust treatise on handling radio signals in a purely mathematical setting.

3.4 ANSYS courses (1cp)

A myriad of small courses will be taken from ANSYS to provide the sufficient background to conduct this research project. Each course consists of training modules by video, concluding with a quiz to consolidate understanding on each subject. They each have a duration of 2 hours, therefore constituting as a a single credit point.

- Electromagnetic wave propagation (https://innovationspace.ansys.com/product/electromagnetic-wave-propagation/)

- Theory of electromagnetic radiation (https://innovationspace.ansys.com/product/theory-of-electromagnetic-radiation/)

- Electromagnetic plane waves (https://innovationspace.ansys.com/product/electromagnetic-plane-waves/)

- Design of the half-wave dipole (https://innovationspace.ansys.com/product/design-of-the-half-wave-dipole/)

- Frequency characteristics of the half-wave dipole (https://innovationspace.ansys.com/product/frequency-characteristics-of-the-half-wave-dipole/)

The first three courses relate to physics, specifically electromagnetism. Such courses are necessary to develop a physical understanding of radio technology that will ultimately be reduced to mathematical formalism. For instance, an understanding how RF waves are generated, and the physical characteristics of propagation (speed, direction, attenuation etc.) are all properties that the project desired to model mathematically; these courses demonstrate the physical understanding of such principles that my project seeks to demonstrate by direct derivations from Maxwell's equations.
The last two courses correspond to antenna theory; the theory in Nahin (2025) avoids discussing the more intricate details of antennae theory, therefore in order to give a mathematical representation for antennae, it will be necessary to seek information beyond the scope of Nahin (2025). These courses specifically look at the half-wave dipole; an antenna whose simple design principles make it feasible to study in a mathematical setting.

3.5 Wolfram U; Introduction to Differential equations (1cp)

https://www.wolfram.com/wolfram-u/courses/mathematics/introduction-to-differential-equations/
Although Fourier analysis and digital signal processing is the mathematical formulation, the construction of radio emitting circuits brings several differential equations into discussion, such as the famous Van Der Pol equation. They hold importance in calculating necessary impedance and transfer functions associated with radio circuits, and while numerical solutions are useful to the engineer, a rigorous mathematical approach instead requires analytic solutions and qualitative theorems on these equations. Because of this necessity, the ability to solve various differential equations that appear in the study of radio is a mathematical necessity of the project along with Fourier analysis and Maxwell's equations.

3.6 Literature

The research questions of this project are motivated by two works of literature in particular; 'Fourier and Laplace transforms' and 'The Mathematical Radio'. Particularly, the first research question is motivated by Beerends (2006) and the second by Nahin (2025).
Although the literature below also contains overlap with the coursework, literature is of particular importance due to the open questions and conjectures proposed in the book, particularly by Nahin (2025).

3.7 Fourier and Laplace transforms

A book translated into English from Dutch, Beerends (2006) lays down the fundamentals of Fourier analysis in a pure mathematics setting with chapters applying knowledge to control theory. It discusses the theory of Fourier series, Fourier integrals, and their extension to the theory of distributions, which will constitute as the fundamental building blocks of radio signals. The book will serve as a 'pure mathematical' basis of knowledge, acting as a supplementary resource to complement the digital signal processing module. Although it does demonstrate applications of Fourier analysis to systems in the sense of control theory, it lacks direct application of Fourier analysis to signal processing and radio technology in particular; hence with regards to connecting mathematics to radio technology, the project will require other resources.

3.8 The Mathematical Radio

The motivation of this book was to convince to a 'ressurected Hardy' (a staunch pure mathematician) the mathematical aesthetic of the applied mathematics of radio technology. Because of this, the book places great emphasis on attempting to demonstrate physical results by means of pure mathematical approaches, and proposes several open questions that could potentially be resolved by such means.
The proposed project is essentially an extension upon this work; resolving the questions it poses, exploring more sophisticated mathematical techniques and using them to pose and answer new questions.

3.9 Plan matrix

To summarise the research plan, a related 'plan matrix' has been listed below, summarising the proposed courses and literature to be undertaken and read in preparation of the topic.

Component	Origin	Contents	Credit Points
TRM Compulsory Module	UTS	Research design principles including identifying research gaps, selecting appropriate research methodologies, distinguishing between qualitative and quantitative approaches, structuring research plans, and evaluating research outcomes relevant to the research methodology.	1
Digital Signal Processing: Fundamentals	Coursera	Mathematical foundations of digital signal processing including discrete-time signals and systems, Fourier series and discrete Fourier transforms, sampling theory, spectral analysis, and filtering. Emphasis on the role of Fourier analysis in representing and analysing signals used in communication systems.	3
Electromagnetics and Antenna Theory Modules	ANSYS Innovation Space	Series of modules covering electromagnetic wave propagation, radiation from antennas, plane wave solutions of Maxwell's equations, and antenna design concepts such as the half-wave dipole. These modules provide the physical basis reqired to model radio technology mathematically.	1
Introduction to Differential Equations	Wolfram U	Ordinary differential equations relevant to engineering systems including first- and second-order differential equations, linear systems, stability analysis, and qualitative behaviour of dynamical systems. Applications include oscillator models and circuit dynamics relevant to radio transmitters.	1
Fourier and Laplace Transforms	Beerends et al. (2006)	Mathematical treatment of Fourier series, Fourier transforms, Laplace transforms, convolution, and distribution theory. Serves as the mathematical foundation for analysing signals and solving differential equations arising in signal processing.	Literature Study
The Mathematical Radio	Nahin (2025)	Mathematical analysis of radio systems including amplitude modulation (AM), frequency modulation (FM), single-sideband modulation, and Fourier-based signal analysis. The book also presents open problems and mathematical questions relating to radio technology that motivate the project.	Literature Study

Table 1: Summary of coursework and literature supporting the thesis research.

4   References

Beerends, R. J., & Al, E. (2006). Fourier and Laplace transforms. Cambridge University Press.
Nahin, P. J. (2025). The Mathematical Radio. Princeton University Press.